Mathematical Beauty in Rome: A Study-Abroad Program for STEM Students

Joseph Pasquale
University of California, San Diego, pasquale@cs.ucsd.edu

Abstract – **Mathematical Beauty in Rome is a five-week summer study-abroad program at UCSD designed specifically for college undergraduates in STEM majors; it has been offered since 2008. Its subject matter is the architectural geometry and structural engineering of great monuments in Rome. The program is comprised of two courses, one that takes place in the classroom and the other that takes place at various sites, using Rome as a living laboratory. Some of the major sites that are studied include the Colosseum, the Pantheon, St. Peter's, and the Aqua Claudia Aqueduct. There is a one-week excursion to Florence to study Brunelleschi's dome and to Pisa to study the Leaning Tower. Works of art such as Raphael's School of Athens are also studied for their use of mathematical perspective. The paper presents the motivations and goals of the program, pedagogical methods based on experiential learning, an example of course content on the Colosseum's ground plan geometry, cultural aspects, assessments and evaluations.**

Index Terms – STEM, study-abroad, experiential learning, geometry, structural engineering, Roman architecture

INTRODUCTION

Mathematical Beauty in Rome (http://mathinrome.ucsd.edu) is a study-abroad program for undergraduates offered since 2008 at the University of California, San Diego (UCSD), and is part of the UCSD Global Seminar series. The program takes place in Rome, Italy, for five weeks during the summer. The program is comprised of two official 4-unit courses (i.e., each a standard full-time 10-week/quarter course, run in double time) that together explore classical Roman architecture from a mathematical and engineering point of view, focusing on the geometries and structural principles of the Colosseum and the Pantheon, amongst many others. The program also includes a one-week excursion to Florence to study Brunelleschi's dome and to Pisa to study the Leaning Tower.

The program seeks to synthesize aspects of mathematics (mostly geometry), engineering (mostly structural), architecture (Imperial Roman, Renaissance, and Baroque), and philosophy (ancient Greek). Figure I shows these subjects and their interrelatedness, reflecting how the ancient Romans treated them with philosophy placed in the center, emphasizing the deep influence that Greek notions of beauty (including and especially *mathematical beauty*) had on ancient Roman thinking. Students learn about these topics in the classroom as well as on location at various sites, considering questions such as:

- What geometrical forms were used in the design of the Colosseum, the Pantheon, Brunelleschi's dome, etc., and why?
- How do the arches and domes that comprise these structures physically work (i.e., engineering statics), and how are they analyzed for stability?
- More generally, what engineering principles enabled some of the world's greatest structures to last for two millennia, and what kind of geometry led to their lasting beauty?

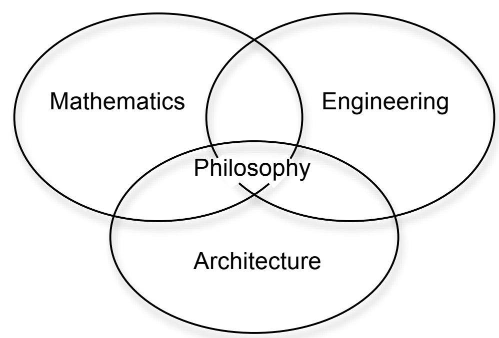

FIGURE I
BROAD TOPIC AREAS OF MATHEMATICAL BEAUTY IN ROME

The questions are not limited to architecture, but include art:

- How is mathematical perspective achieved in Raphael's masterpiece, *The School of Athens*?
- How are the proportions of Michelangelo's *David* geometrically modified to be more visually appealing from the viewer's perspective?

These questions are dealt with in a mathematically rigorous manner (students are expected to have taken an undergraduate Calculus series for Engineers and Scientists and to have background in Euclidean geometry). In the classroom, the students learn the theory to be able to answer these questions, and at the various sites, they see how the theory is applied in practice by carrying out experiments. This interplay is key, and will be a recurring theme throughout this paper. It is one thing to study the geometrical and structural properties of a dome inside a classroom, and it is quite another to be inside the Pantheon, or inside (literally, within the inner and outer walls) of Brunelleschi's dome in Florence, making observations and measurements.

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Since the program's inception in 2008, 110 students have graduated from the program. The typical class size is 15-20 students, enough to make the program viable, yet small enough to promote group interaction during site visits in tightly-spaced areas. The program draws from a diverse group of students, of which 54% are women, and also of which 25% are underrepresented minorities.

The program is designed to appeal to all STEM majors, and virtually all areas of engineering, mathematics, and sciences with offered degrees at UCSD are represented, along with inter-disciplinary STEM-related areas and some non-STEM areas. A breakdown is shown in Table I.

TABLE I
DISTRIBUTION OF UNDERGRADUATE MAJORS ENROLLED IN MATHEMATICAL BEAUTY IN ROME, 2008-2015 (SPECIFIC MAJORS IN ORDER OF POPULARITY)

%	Area	Specific Majors
50.0	Engineering	Structural, Computer Science, Electrical, Mechanical, Biological, Aerospace, Chemical, Environmental, Nano
24.6	Mathematics	Math, Applied, Math/Econ, Math Education
10.5	Sciences	Biology, Physics, Biochemistry, Chemistry
7.0	Inter-Disciplinary	Urban Studies & Planning, Management Sci, Cognitive Sci, Visual Arts
7.9	Other	Intl. Studies, Economics, Political Sci, Human Development, Psychology, Chinese Studies

The advantages of having a wide variety of STEM majors are numerous, and will be discussed further in the next section. A disadvantage is that because the program has a single home department, its courses typically do not fulfill requirements of other departments. Despite this (and perhaps because of this), the students who apply do so primarily because of their passion for the subject matter and desire to study that material in a marvelous environment abroad, and so they tend to be more focused and serious.

MOTIVATION

The author's motivation for designing Mathematical Beauty in Rome was a desire to offer a study-abroad option for STEM majors at UCSD. In 2007, a new international education program, the UCSD Global Seminar Program, had just been announced, whereby UCSD faculty could propose two official 4-unit (i.e., full-time full-quarter) UCSD courses to be taught abroad, and that would be attended by UCSD students. In other words, these are standard UCSD courses, taught by UCSD professors, for UCSD students (but also open to students from other UC campuses, and indeed even students from other universities can apply); the only difference is that the courses take place abroad. The courses would have to be ones that would arguably benefit by being taught in a foreign location. As there were no such existing STEM courses that fit this criteria (at least not as well as "Literature, Art, and Film in Paris," "Modern Greece," "The Italian Renaissance"), the author designed a pair of courses from scratch that focused on mathematics and engineering.

Students who have had study-abroad experiences typically describe them as "life-changing." Unfortunately, STEM students typically do not study abroad for various reasons, one being that there are not many programs targeted for them, i.e., whose subject matter is specifically in STEM fields (of the 10-15 UCSD Global Seminars offered yearly since 2008, the only one in STEM has been Mathematical Beauty in Rome). In the most recent survey by the Institute of International Education (IIE), only about 69,000 students studied abroad in STEM fields in 2013/14. [1] This is small relative to the roughly half million of students that received bachelor's degrees (never mind the more than 3 million that are simply enrolled) in STEM fields in 2011-12 (latest data available) as reported by the National Center for Education Statistics. [2] (A positive trend is that of all students studying abroad, STEM has increased its share from 16% in 2003/04 to 23% in 2013/14. [1])

Mathematical Beauty in Rome is one effort in seeking to remedy this problem, with the goal of presenting challenging STEM material – architectural geometry and structural engineering – "in context," i.e., taking advantage of the local environment that is Rome and using the city as a living laboratory.

Why is study abroad of special value to STEM students? Simply put, having global experience is important. In "The Newport Declaration," an influential report on Globalization of Engineering Education by 23 distinguished engineering educators from top universities, it is emphasized that "Global experience is increasingly perceived as essential to career success in science, technology, engineering, and math." [3] Furthermore, their primary recommendation directs educators, administrators, and policy makers "to take deliberate and immediate steps to integrate global education into the engineering curriculum to impact all students, recognizing global competency as one of the highest priorities for their graduates." [3]

With an increasingly global economy, it is important that STEM graduates be able to work across national boundaries and with international agencies. International experience promotes understanding of different cultures and reduces the barriers they present. This is being recognized by universities, and some, one of the first being Rensselaer Polytechnic Institute, are making international experience a mandatory part of the engineering curriculum. [4]

GOALS

Mathematical Beauty in Rome was designed with the following three goals for students:

1. To promote understanding of the concept of mathematical beauty

On day one of class in Rome, students are first asked: What is *beauty*? (This is a surprise to most STEM students as this is not a question they expect to be asked in their typical courses). This then proceeds into a discussion of how Romans perceived beauty. This leads to the more focused question: What is meant by *mathematical beauty*? Students in STEM majors are so often immersed in the mechanics and rigor of their subject that they often do not appreciate the beauty of the concepts in their fields. To this day, the proof of the irrationality of √2 in Euclid's Elements [5] is recognized as a gem of mathematical beauty, and we discuss why. Such results, especially in geometry, go back to the times of ancient Rome. Finally, the question is further refined to: Where might one find *mathematical beauty in Rome*? This naturally leads to a discussion of the wonderful sites to be studied over the next five weeks.

2. To show that basic results in geometry and engineering permeate Roman architecture and structures

The Romans did not design in ad hoc or haphazard ways. There is design based on Euclidean geometry everywhere, be it in the small-scale design of a simple spiral volute at the top of an ionic column, to the grand-scale oval ground plans based on piecewise polycentric circular arcs of the Colosseum and St. Peter's piazza, or the embodiment of Archimedes' geometrical construct of a sphere-in-a-cylinder in Rome's Pantheon. And while modern notions of structural engineering were not known to the Romans, it is amazing how close their geometrically derived structures match what modern structural engineering tells us is optimal in efficiency and stability.

Students are encouraged to see Rome beyond that of the everyday tourist, as one with "mathematical eyes," seeing geometrical constructions in architecture and valuing the power of geometry in engineering design. There is also an element of detective work in trying to understand geometric problems in terms of what Romans knew about geometry (and therefore what they likely used) that is evident from such observations. For the most part, the ancient Romans did not leave any plans behind, and so a fun part of the students' experience is a bit of mathematical and engineering "archaeology," i.e., seeking to hypothesize about the likely plans (e.g., geometric constructions) of Roman structures based on the remains that we can observe today.

3. To broaden horizons, both academically and culturally

Students in STEM areas are often viewed as "narrow," and not without reason given the focus required for their specific areas of study, which are often highly technical and specialized. Exposing students to the mathematics and engineering of a very different age, both of which were highly effective, and in the context of a very different culture, is certainly a broadening experience. A recent report by the National Research Council (NRC) titled "The Mathematical Sciences in 2025" emphasized the importance of promoting broadness in STEM education: that students should be "knowledgeable across a broad range of the discipline, beyond their own area(s) of expertise," to be able to "communicate well with researchers in other disciplines," and to "understand the role of the mathematical sciences in the wider world of science, engineering, …". [6]

For students who come from a wide range of STEM disciplines, to share a common learning and living experience, and to have a chance to *learn from each other*, is in line with this broadening directive. The on-site laboratories and homeworks given to the students are expected to be done collaboratively, and naturally lead to a development of mental tools that span and connect disciplines (and in fact, promoting such "connections" is especially emphasized in the aforementioned NRC report [6]). Beyond academics, a high degree of collaboration is needed just in navigating the city of Rome, using the transportation system, planning group meals and shopping for food in the various markets, planning trips on weekends, etc. Watching analytically-minded students from different areas, each bringing their specialty to the table, is especially fun and interesting for the instructor to observe.

METHODS

The academic portion of the program consists of two courses:

- CSE 4GS: Mathematical Beauty in Rome (Classroom)
- CSE 6GS: Mathematical Laboratory in Rome (On-Site)

The meetings of these courses are interleaved, and build on each other. The first course takes place in the classroom (at facilities provided by the American University in Rome), and is partly lecture and partly discussion. Here students learn theoretical concepts in both geometry and structural engineering – how to do certain geometrical constructions, how to analyze their complexity, static analysis of structures (including arches and domes), how to determine stability – which form the basis of their required knowledge to be able to analytically understand the structures to be visited.

The second course takes place on location at the various sites. Examples include: Aqua Claudia Aqueduct, Roman and Imperial Forums, Palatine Hill, Colosseum, Pantheon, Campidoglio, Baths of Caracalla, St. Peter's basilica, Via Appia Antica, Castel Sant'Angelo, church of Sant'Ignazio; full-day excursions outside Rome to Hadrian's Villa and Ostia; a one-week excursion to Pisa (Campo dei Miracoli: Leaning Tower, Duomo, Baptistry, Camposanto), Florence (Santa Maria del Fiore, Baptistry, Campanile; Santa Maria Novella; Santa Croce). At each site, one or more specific structures are studied in detail. For example, the visits to St. Peter's in Rome and Santa Maria del Fiore (the "Duomo") in Florence focus especially on their domes, going inside them, climbing to the top, etc. Being able to observe, close up, the pattern of brickwork in Brunelleschi's dome is an invaluable experience, especially after having studied the design rationale and structural properties derived from its intriguing and celebrated "herringbone" pattern.

These two types of experience – classroom and on-site – are further supplemented by visits to major museums: Capitoline and Vatican in Rome; Civilta Romana in EUR; Opera Duomo, Uffizi, Accademia, and Galilean (Science) in Florence. These visits provide important cultural and historical contexts for the more technical material learned in the courses (as well as technical contexts, e.g., actual tools and machinery used by Brunelleschi).

The pedagogical method used in Mathematical Beauty in Rome is based on "learning by doing" and "experiential learning." These are not new ideas (though we do apply and extend them in unique ways, discussed below), even going back to time of ancient Rome: in Aristotle's Nichomachean Ethics, he writes "for the things we have to learn before we can do them, we learn by doing them" [7]. The modern theory of experiential learning is rooted in the ideas of educational philosopher John Dewey, emphasizing the role of experience, experimentation, interaction with the environment, reflection, and purposeful learning [8]. A key idea is that the teacher judiciously chooses experiences to allow students to learn from them on their own. Kolb developed the "Experiential Learning Model," (ELT) the elements of which involve: (1) concrete experience; (2) reflective observation; (3) abstract conceptualization; (4) active experimentation. [9] In more recent work, Passarelli and Kolb argue that ELT can be successfully applied to study-abroad, emphasizing that "Attention must be paid to designing a learning experience that helps students fully absorb and integrate their experiences at increasing levels of complexity." [10]. Others have also explored the application of experiential education to study-abroad. [11][12]

We apply and extend these ideas as follows. In the classroom, the students learn certain concepts, all of which are preparaton for carefully set-up experiences that will take place at a particular site. These concepts are purposefully explained in the abstract, e.g., the geometric construction of a certain type of curve. Then, on site, the key activity of what we call "prepared discovery" (i.e., prepared by the instructor for discovery by the student) occurs, where the student has a concrete experience in which they encounter the abstract concept, e.g., that the curve of the Colosseum seems to be similar to what was discussed in class. Whether this is indeed the case can be verified by certain tell-tale signs or clues, which the student also learned and so then tries to verify, e.g., by walking around the inside the Colosseum and seeking locations where the alignment of certain structures should occur if indeed the curve is what was hypothesized. Upon discovering these signs, there is a deep feeling of delight and accomplishment by the student, and a lesson the student will not forget.

Classroom concepts are reinforced with homework, typically of geometric constructions (carried out using Geometer's Sketchpad software [13]) and problems in engineering statics. On-site activities are organized as laboratory projects, involving observations, measurements, experiments, etc. Both homeworks and labs are done in collaborative groups, and typically will involve iterative submissions where feedback is given until the submissions are correct. Collaboration encourages students to help each other to learn the material, promoting camaraderie (especially important as they are a somewhat isolated group living in a foreign country, and so working in teams academically translates to positive reliance on each other in living matters). Finally, there are two in-classroom exams (one per course); consequently, despite working in groups, each student understands they are personally responsible for learning the material as all the exams are done individually.

AN EXAMPLE OF COURSE CONTENT

To give a more concrete idea of the methodology and the kind of technical content covered, we present an example in some depth. This is followed by a listing of other topics with brief descriptions to illustrate the scope of the courses. This example will give a better idea of what students actually learn, and will illustrate the interplay between theory and practice discussed above, emphasizing the importance of supplementating classroom discussion with site visits and how they can effectively shape the learning experience.

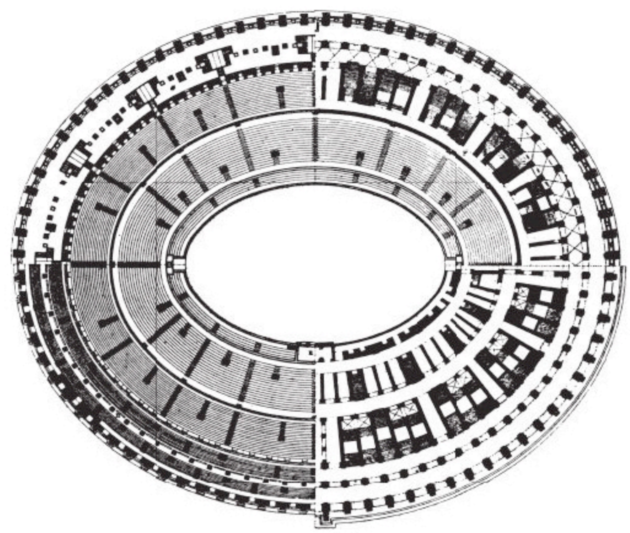

FIGURE II
GROUND PLAN OF THE COLOSSEUM IN ROME [14]

I. Geometry of the Colosseum

A major topic of study is the Colosseum, and more generally, the Roman amphitheater, of which the Colosseum (originally called the *Flavian Amphitheater*) is just one, albeit the most famous, example. The Roman amphitheater is especially interesting because it is uniquely Roman in its architecture, its engineering, and its place in ancient Roman culture. Many Roman cities had their own amphitheaters, and hundreds were built. Unfortunately, there are no contemporaneous written records on how they were designed or constructed. In particular, Vitruvius, who wrote his *De Architectura libri decum* (The Ten Books on Architecture) [15], the oldest extant book on the theory of architecture written around 15 BCE, is silent on this topic. However, much has been written in the last 300 years, and in fact, the last twenty years has seen much research activity on the design and construction of Roman amphitheaters [16]-[17]. Because there are so many of them, and because they were built over a period of time (roughly 200 years), researchers have been able to develop a theory of design by studying how they evolved.

Roman amphitheater design reached its pinnacle, in complexity and size, with the Colosseum. It was built during the years 70-80 CE by the emperors Vespasian and his son Titus, and had a seating capacity of 50,000 or more (comparable to that of modern stadiums).

As shown in Fig. II, the Colosseum's ground plan has an "elliptical" design, with dimensions 189 m in length and 156 m in width (its height is 48 m). Two questions that have baffled researchers, even to current times, are, "Is it really an ellipse?" and "How exactly did the Romans generate it?"

When students are asked the first question, the typical response is that it is indeed an ellipse. And why not: it looks like an ellipse, and by placing the foci in just the right places, a true ellipse will in fact fit quite well (at least for the outer perimeter) [18]. As to the question of how it might have been generated, the Romans were certainly familiar with what we now call "the gardener's method," where two stakes are placed at the foci connected by a rope such that when held taut, the rope forms two legs of a triangle with the third leg being the length between the stakes.

This is where theory and practice diverge. First, there is the practical problem with using such a long rope and preventing it from stretching and thus distorting the shape (some researchers have suggested that perhaps a chain was used). But the more fundamental problem is a mathematical one. What is evident from the ground plan in Fig. II is that the rows of seats along each circumference (as well as the walkways) are parallel (as is true of any stadium; the distance between rows of seats and the width of the walkways are constant). And yet, it is a mathematical property of concentric ellipses that they cannot be drawn parallel [18], i.e., the points of the inner ellipse will not be at a fixed normal distance to those of the outer ellipse, regardless of how the foci of each are placed, something most students learn for the first time.

Thus, either the Romans made some kind of ad hoc adjustment, or perhaps they had some other method. The Romans were certainly well aware of Euclidean geometry and straightedge-and-compass constructions. Are there constructions that generate shapes that look like ellipses?

The answer is yes, and one source from which we know this is the work of Sebastiano Serlio who lived during the late 15th/early 16th centuries. In one of his books in *Tutte l'opere d'architettura et prospetiva* (All the works of architecture and perspective) [19], he describes a straightedge-and-compass construction that generates ovals, sometimes now referred to as "carpenter's ovals," as shown in Fig. III. Serlio lived more than 1400 years after the construction of the Colosseum, and so what evidence is there that the Romans knew of this method? Serlio's book was not necessarily a work describing new constructions, but rather a compilation of what was known at the time, and since it is the earliest work we have of such constructions, perhaps some go back to ancient Roman times. This serves as a good lesson for students that what we know about the ancient world, including mathematics, is often based on what is found in much later works.

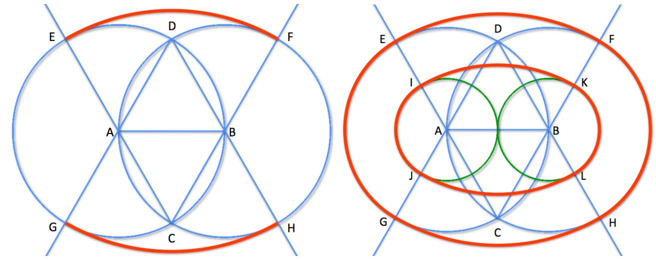

FIGURE III

STRAIGHTEDGE-AND-COMPASS CONSTRUCTION FOR OVALS USING SERLIO'S METHOD. REFERRING TO THE LEFT CONSTRUCTION, STEP 1: DRAW LINE AB; STEP 2: DRAW CIRCLE A, B (I.E., CENTERED AT A WITH RADIUS AB); STEP 3: DRAW CIRCLE B, A; STEP 4: LOCATE INTERSECTIONS C AND D; STEP 5: DRAW RAYS CA, CB, DA, DB; STEP 6: LOCATE INTERSECTIONS E, F, G, H; STEP 7: DRAW ARCS EF (OF CIRCLE C, E) AND GH (OF CIRCLE D, G). FOR INNER OVAL, USE SAME FOCI A AND B BUT SMALLER RADIUS AI, AS SHOWN IN RIGHT CONSTRUCTION.

An important part of this course is that students become aware of these works, e.g., those of Vitruvius, Serlio, and others, and accepting the fact that the ancient Romans left little documentation behind as to how they did what they did, that they try to come up with theories based on the evidence, i.e., what they observe as part of the existing structures, to see if they can solve these puzzles.

Applying this to the Colosseum, by carefully studying the ground plan and the layout of the seats, especially their radial design, a pattern does emerge. If lines are drawn along the radii formed by back-to-back seats and radial walls, they lend credence to the idea that perhaps the ancient Romans were in fact aware of Serlio's straightedge-and-compass constructions for ovals.[1]

To further confirm this theory, upon visiting the Colosseum, the students make observations (and take photographs) at various key locations, looking to see if the radial walls on the other side of the Colosseum happen to align (this corresponds to the radii meeting at a vertex of the construction). This method of detective work and discovery is extremely effective in learning about basic geometry and Roman ingenuity (i.e., doing so much with so little, a good lesson for modern day students who have so much computing power at their fingertips).

In this sample lesson, the interplay of theory and observation are key. Students come to understand that the constructions they learn are not solely abstractions that have little bearing on reality. In fact, they learn that making careful observations can inform their intuitions in developing and understanding the theory, and vice versa.

That these lessons can be given in the context of what is a rich cultural tradition that is ancient Rome makes them especially memorable for students. There is always an interesting story about what an emporer envisioned versus what was actually achievable (in terms of design or

[1] Cozzo posited that the ancient Romans may have used a more elaborate construction that is an extension of Serlio's and that provides a more precise fit. [20] This advanced construction is often used as a final exam problem.

construction), or how architects kept their ideas and methods secret (in contrast to our modern ideas of information dissemination and how credit is determined). A more general and important lesson is about how education (albeit for the upper classes) was perceived at the time, especially that knowledge in *many* areas, spanning art, science, and philosophy, was supremely valued. Vitruvius is especially instructive on this. Paraphrasing from Chapter 1 of *De Architectura* on "The Education of the Architect" [15]:

> The architect's expertise is enhanced by many disciplines born both of practice and of reasoning. An architect should understand letters, draftsmanship, geometry, optics, arithmetic, history, philosophy, physiology, music , medicine, law, astronomy, …

For each area of knowledge, he discusses why it is important and how it relates to the others. In Mathematical Beauty in Rome, this viewpoint is presented right from the beginning and is a theme that runs throughout the entire program.

II. Topics Covered

To complete this section on course content, we present a list that summarizes some of the many other topics covered:

- Vitruvius on "What is architecture" and that "good building" should satisfy "firmitatis, utilitatis, and venustatis" (durability, utility, and beauty)
- Influences on the Romans by the Greeks and the Etruscans, and how the Romans synthesized these ideas and used them on a grand scale
- Basic concepts in architecture: orders, building types, emphasis on mathematical proportions, periods, styles
- Broad history of Rome, to provide historical context
- Basics of engineering statics, enough to derive how arches and domes work
- The Roman arch, its structural properties and analysis, discussing the development of arched structures of different shapes (from semi-circular to pointed, etc.), and deriving the ideal arch based on the catenary
- The Roman dome, building on the description and analysis of the Roman arch, and while a Roman dome is indeed simply a Roman arch rotated about its central axis, there are important differences in its structural properties and how the forces distribute
- In-depth analysis of the dome of the Pantheon, with on-site activities (similar to those described for the Colosseum)
- In-depth analysis of the Brunelleschi's dome in Florence with on-site activities that take place inside, and an important lesson of cultural heritage in how Brunelleschi studied the Pantheon in coming up with his then revolutionary design
- Geometrical proportions of Bramante's Tempietto and its influence on later designs (especially that of St. Peter's basilica); on-site activities include making measurements and validating proportions as to how they fit in architectural theory developed in the Renaissance
- In-depth analysis of Michelangelo's dome of St. Peter's basilica with on-site activities that take place inside the dome, and how Michelangelo learned from the domes of the Pantheon and that of Brunelleschi
- Geometry of spiral structures and their manifestations in Roman architecture (ionic volutes, the lantern on the dome of Sant'Ivo alla Sapienza by Borromini)
- Stability of Pisa's leaning tower (and while in Pisa, a visit to the statue of Fibonacci with an on-site lesson on the golden ratio)
- Evolution of architecture at Hadrian's Villa, where one can see many experiments in dome design
- Theory of mathematical perspective as developed by Alberti and its use in works that the students see, including Masaccio's *Santa Trinità* (Santa Maria Novella, Florence) and Raphael's *School of Athens* (Vatican Museum, Rome)

CULTURAL ASPECTS OF THE PROGRAM

The program is not just about academics. Indeed, an equally important part of the program is what students experience outside the classroom. Learning about a different culture, learning to be flexible, learning how to get by in a foreign land, learning how to rely on your classmates because one cannot do it alone, these are all important lessons that provide a unique opportunity for personal growth.

Rather than viewing these opportunities as ones that compete with the program's academics, it is advisable to find ways of integrating them. For example, presenting the material in a historical and cultural context makes it not only more interesting but also more relevant to when students are on their own exploring in the local environment. When visiting, for example, the Aqua Claudia aqueduct, which is some distance outside the city, experiencing the heat of the Roman sun in July brings home the immense importance the ancient Roman's placed on bringing fresh cold water into the city – to drink, to cool off, and to keep clean – so much so that they built immense aqueducts (that included tunnelling) to transport the water from hills many miles away.

To promote unity and camaraderie amongst the students, there are many group meals together. This also provides an opportunity for students to learn about Italian food and its many types (e.g., regional) and the value of spending time together during meals, both of which are very important in Italian culture. Keeping the students together in a study-abroad program is very important, as the last things a student should feel are isolation and alienation; on the contrary, it should be one of their most memorable positive life experiences.

As discussed above, the lesson plans have homework assignments that require exploration and observation. Rather than asking students to carry these out during some fixed amount of time within some fixed interval, they are encouraged to integrate them as part of their normal explorations in Rome. For example, students learn in class that there is no exact geometrical construction (limited to straightedge and compass) for a regular heptagon, and so, perhaps seven-sided figures are a rarity in Roman architecture. An ongoing activity is to look for seven-sided figures, take photos of them, and bring them in to class for discussion. While rare, they do exist, and what is especially interesting is that there is usually a good reason for using a figure that required a construction that was both approximate and more complex than that of the others. A good example is the use of 28 in the 28x5 matrix of coffers in the Pantheon's dome, requiring the laying out of 7 equally spaced points in a circle (and then halving the intervening distances twice). This typically leads to interesting discussions for why 28 was used for the Pantheon, when many other domes used coffered designs with numbers of sides more easily constructed, 32 being a common one.

ASSESSMENTS AND EVALUATIONS

For every offering of the program (since 2008), literally every single student has done extremely well. There are some good reasons for this:

- As indicated in the section on Methods, all homeworks and laboratory projects are done iteratively with intervening feedback from the instructor, until they are completely correct. Students have an incentive to complete their work and submit it early so they can benefit from this feedback. They learn from their mistakes, and ultimately learn the material well. Being able to collaborate and learn from their peers is also very helpful, as they tend to discuss the homework problems whenever they are together, even on their "off" time just exploring Rome.
- As mentioned in the Introduction, since the courses do not fulfill any standard university requirements for any specific major, the only students who enroll are ones who are highly motivated, serious and passionate about the subject matter for its own sake. Furthermore, the students who enroll are typically high achievers, and this is exemplified in their continued high performance during the program.
- Not insignificant is the fact that the entire program costs roughly $10,000 (when *everything* – tuition, housing, meals, airfare, trains and local transportation, entrance fees to all sites, etc. – is taken into account; while this is a high cost, almost all students get grants and loans from various organizations, making the cost manageable even for students of limited means.) To make this major investment, only to then not take the experience seriously, goes against the fiber of most students.

The end-of-program reviews filled out by students have been overwhelmingly positive. The reviews include a series of statements to which they can state their level of agreement on a scale of 1-5, where 5 = Strongly Agree, 4 = Agree, 3 = Neither agree nor disagree, 2 = Disagree, and 1 = Strongly Disagree. The average responses to three of the key and most important ones are shown in Table II (all the other statements had similarly highly positive response levels).

TABLE II
STUDENT RESPONSES IN END-OF-PROGRAM REVIEWS
MATHEMATICAL BEAUTY IN ROME, 2008-2015

Statement	Level of Agreement (5 = Strongly Agree, 1 = Strongly Disagree)
"The course material is intellectually stimulating"	4.9
"I learned a great deal from this course"	4.8
"Do you recommend this course"	5.0

Perhaps nothing tells the story better than what students had to say in their own words. A representative sample of the comments collected from the reviews are shown in Table III.

TABLE III
COMMENTS FROM STUDENTS IN END-OF-PROGRAM REVIEWS
MATHEMATICAL BEAUTY IN ROME, 2008-2015

Type	Representative Comments
Life changing experience	• "This has been the best, most life changing experience of my life." • "Everything about this program was amazing. This is definitely a life changing experience that every student should try to experience."
Importance of interplay between classroom and site visits (excursions)	• "I found the combination of the lectures and the site visits the most valuable. We learned material in lecture that enabled us to appreciate the sites and we saw things on our site visits that helped us understand and be interested in the lecture material." • "I can't think of too many things that are cooler than learning about a geometric concept in the classroom and then actually getting to see it in ancient Roman architecture." • "It was great to learn about the science behind the construction of ancient Roman architecture before we got to see the concepts first-hand at the sites themselves." • "The group excursions were what made the program complete. The lectures and the visits worked hand in hand by helping us appreciate the beauty of math and of Italy." • "I found the most value in the huge variety of activities and excursions provided by the class. The trips to outside cities, historical sites, and museums were extremely educational and well planned."
Seeing Mathematics Differently	• "The focus of the program provided a unique look at ancient mathematical analysis, which is useful in looking at modern studies in a new way. Having a framework of how mathematics and design began helps me to understand how it works today."
Personal Growth	• "The biggest thing I learned is how important it is to have a broad spectrum of knowledge. It's not enough just to understand math and science, you need to be familiar with different cultures and perspectives in order to work well with others and provide insight." • "Because of this program, I have decided to take more classes for cultural awareness. This program really opened my eyes to the world of academia and what a college level education should mean. Universities are intended to expand one's thought processes to end up with people who are more cultured, think critically, and have knowledge about multiple disciplines. For me, this was an essential realization to extend my time at UCSD." • "I am strongly considering going to grad school to pursue a career in academia because of this program." • "This program has inspired me to be a better person. The city of Rome also showed me great things that have inspired me to try harder in life. It was a life-changing experience."

CONCLUSIONS

Perhaps the most important lesson of having taught this program since 2008 is the value of integrating theory and practice, specifically viewing the geometrical and engineering problems that present themselves in Roman monuments as puzzles that can be solved by careful observation, measurement, and how they relate to theory. An additional important lesson for the students is to observe how much the Romans were able to accomplish with such basic tools, such as simple geometry.

While structural engineering concepts are an important part of the course, the Romans relied solely on geometry (i.e., some shapes work better than others, learned by trial and error, that eventually became architectural building guidelines), as structural engineering was not developed until more recent times. The ability to now look back at these guidelines and compare them to what structural engineering theory tells us, and to reflect on how remarkable it is that the Romans got so much right, is an immensely enlightening experience for students. These points demonstrate the unique value that a study-abroad program designed specifically for STEM students can have for them.

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AUTHOR INFORMATION

Joseph Pasquale, Professor, Department of Computer Science and Engineering, University of California, San Diego.