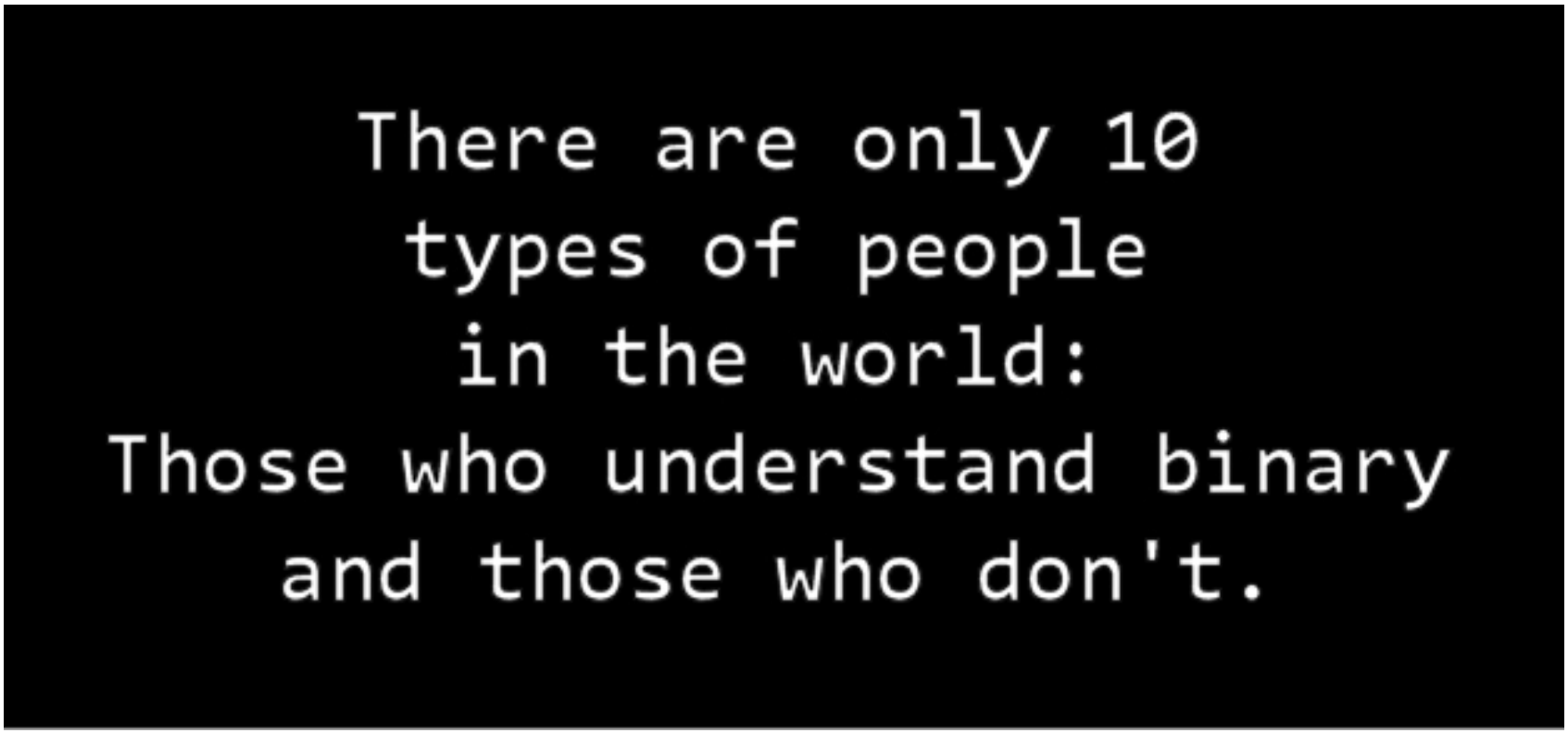
BLOCK CIPHERS
and
KEY-RECOVERY SECURITY

Notation


There are only 10
types of people
in the world:
Those who understand binary
and those who don't.

Notation

$\{0,1\}^n$ is the set of n-bit strings and $\{0,1\}^*$ is the set of all strings of finite length. By ε we denote the empty string.

If S is a set then $|S|$ denotes its size. Example: $|\{0,1\}^2| = 4$.

If x is a string then $|x|$ denotes its length. Example: $|0100| = 4$.

By $x \xleftarrow{\$} S$ we denote picking an element at random from set S and assigning it to x. Thus $\Pr[x = s] = 1/|S|$ for every $s \in S$.

Functions and permutations

By $f : \text{Dom} \to \text{Rng}$ we denote a function taking an input x in the domain Dom and returning an output $f(x)$ in the range Rng.

We say that $f : \text{Dom} \to \text{Rng}$ is a *permutation* if Dom = Rng and there is an *inverse* function $f^{-1} : \text{Rng} \to \text{Dom}$ satisfying

$$\forall x \in \text{Dom} : f^{-1}(f(x)) = x$$

This means f must be one-to-one and onto: for every $y \in \text{Rng}$ there is a unique $x \in \text{Dom}$ such that $f(x) = y$.

Functions and permutations: Example

Consider the following two functions $f : \{0,1\}^2 \to \{0,1\}^2$, where Dom = Rng = $\{0,1\}^2$:

x	00	01	10	11
$f(x)$	01	11	00	10

A permutation

x	00	01	10	11
$f(x)$	01	11	11	10

Not a permutation

x	00	01	10	11
$f^{-1}(x)$	10	00	11	01

Its inverse

Function families and block ciphers

A family of functions (also called a function family) is a two-argument function $F : \text{Keys} \times \text{Dom} \to \text{Rng}$. For $K \in \text{Keys}$ we let $F_K : \text{Dom} \to \text{Rng}$ be defined by

$$\forall x \in \text{Dom} : F_K(x) = F(K, x)$$

Block ciphers

Let E: Keys $\times$ Dom $\to$ Rng be a family of functions. We say that E is a block cipher if

- Rng = Dom
- E_K : Dom $\to$ Dom is a permutation for every key $K \in$ Keys, meaning has an inverse E_K^{-1} : Dom $\to$ Dom
- E, E^{-1} are efficiently computable,

where E^{-1} : Keys $\times$ Dom $\to$ Dom, defined by $E^{-1}(K, y) = E_K^{-1}(y)$, is the inverse block cipher to E.

If Keys $= \{0,1\}^k$ we call k the key length. If Dom $= \{0,1\}^\ell$ we call ℓ the block length.

Block ciphers: Example

This table describes a block cipher E: $\{0,1\}^2 \times \{0,1\}^2 \to \{0,1\}^2$, meaning Keys = Dom = $\{0,1\}^2$. The table entry corresponding to the key in row K and input in column x is $E_K(x)$.

	00	01	10	11
00	00	01	10	11
01	01	00	11	10
10	10	11	00	01
11	11	10	01	00

In this case, the inverse cipher E^{-1}: $\{0,1\}^2 \times \{0,1\}^2 \to \{0,1\}^2$ is given by the same table: the table entry corresponding to the key in row K and output in column y is $E_K^{-1}(y)$. That is, $E^{-1} = E$. Of course this is not always true.

Block Ciphers: Example

Let $\ell = k$ and define $E\colon \{0,1\}^k \times \{0,1\}^\ell \to \{0,1\}^\ell$ by

$$E_K(x) = E(K,x) = K \oplus x$$

Then E_K has inverse E_K^{-1} where

$$E_K^{-1}(y) = K \oplus y$$

Why? Because

$$E_K^{-1}(E_K(x)) = E_K^{-1}(K \oplus x) = K \oplus K \oplus x = x$$

The inverse of block cipher E is the block cipher E^{-1} defined by

$$E^{-1}(K,y) = E_K^{-1}(y) = K \oplus y$$

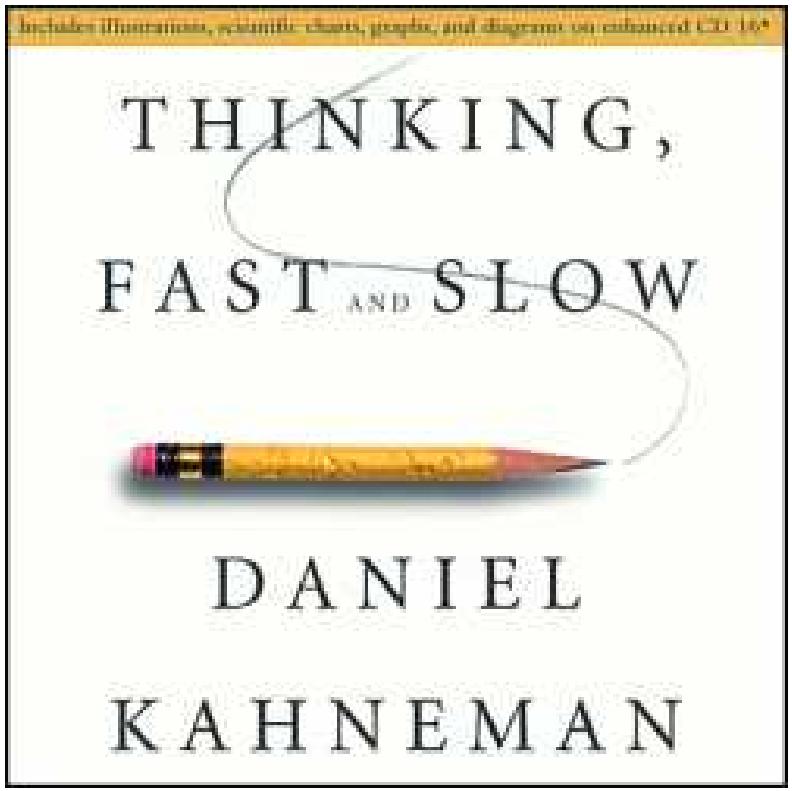
Exercise

Let E: Keys $\times$ Dom $\rightarrow$ Dom be a block cipher. Is E a permutation?

- YES
- NO
- QUESTION DOESN'T MAKE SENSE
- WHO CARES?

This is an exercise in correct mathematical language.

Slow is good



Exercise

Let E: Keys $\times$ Dom $\to$ Dom be a block cipher. Is E a permutation?

How to proceed to answer this: Think slow. Don't jump to a conclusion. Instead:

- Look back at the definition of a block cipher.
- Look back at the definition of a permutation.
- Pattern match these.
- Now make an informed and justified conclusion.

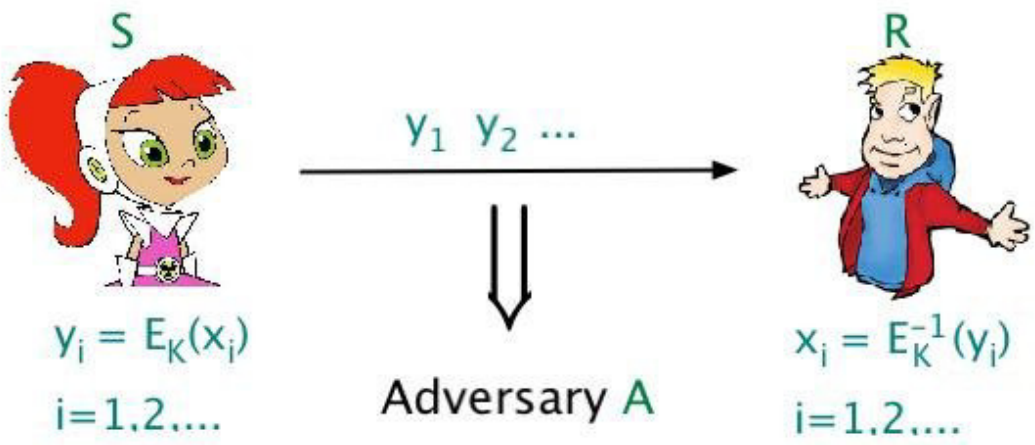
This is an exercise in correct mathematical language.

This is considered a *high-school level* exercise.

Block cipher usage

Let $E\colon \{0,1\}^k \times \{0,1\}^\ell \to \{0,1\}^\ell$ be a block cipher. It is considered public. In typical usage

- $K \xleftarrow{\$} \{0,1\}^k$ is known to parties S, R, but not given to adversary A.
- S, R use E_K for encryption



Leads to security requirements like: Hard to get K from $y_1, y_2, \ldots$; Hard to get x_i from y_i; ...

DES History

1972 – NBS (now NIST) asked for a block cipher for standardization

1974 – IBM designs Lucifer

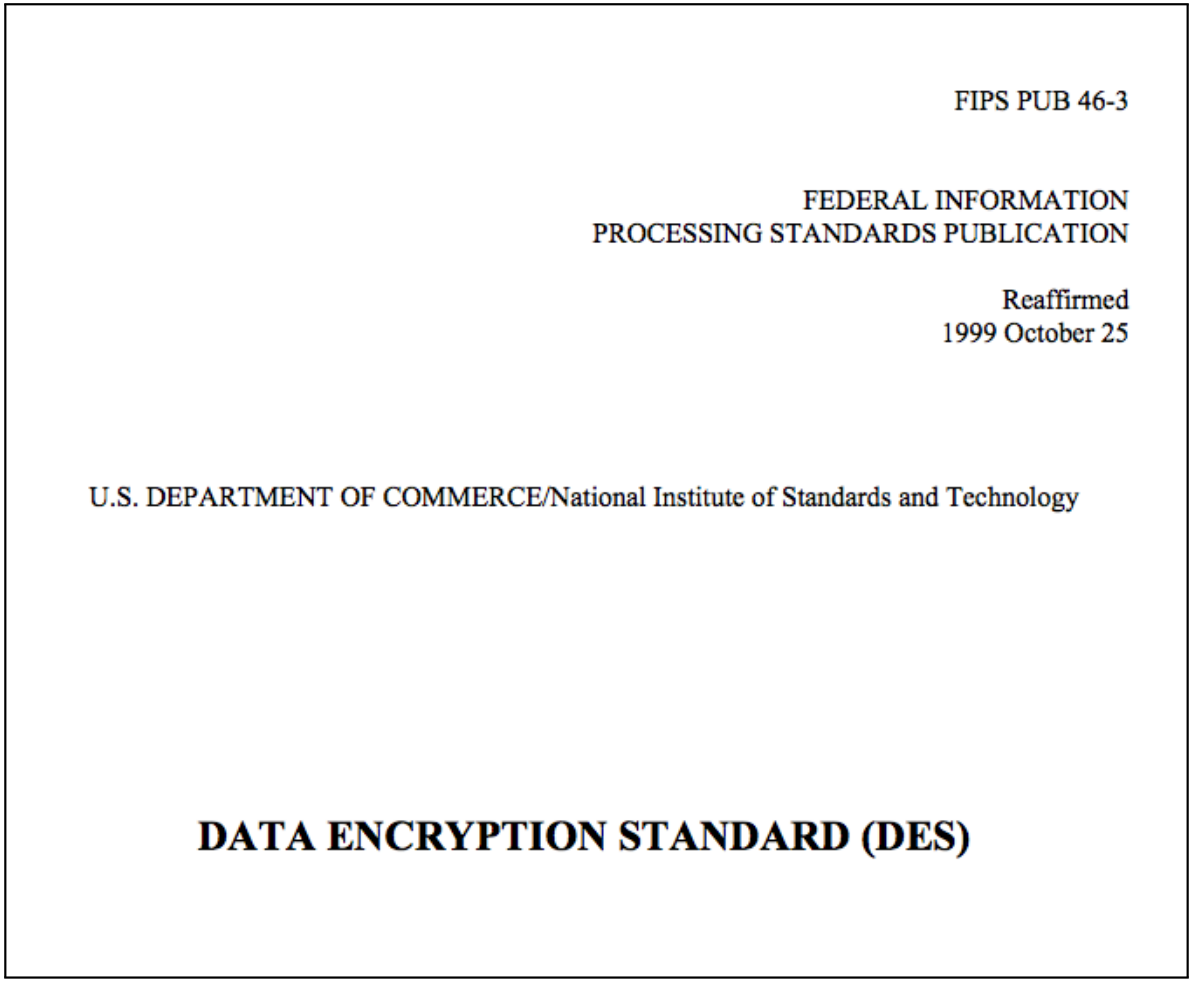
Lucifer eventually evolved into DES.

Widely adopted as a standard including by ANSI and American Bankers association

Used in ATM machines

Replaced (by AES) in 2001.

FIPS DES Standard: Reaffirmed 1999



FIPS PUB 46-3

FEDERAL INFORMATION
PROCESSING STANDARDS PUBLICATION

Reaffirmed
1999 October 25

U.S. DEPARTMENT OF COMMERCE/National Institute of Standards and Technology

DATA ENCRYPTION STANDARD (DES)

DES parameters

Key Length $k = 56$

Block length $\ell = 64$

So,

$$\text{DES}\colon \{0,1\}^{56} \times \{0,1\}^{64} \to \{0,1\}^{64}$$

$$\text{DES}^{-1}\colon \{0,1\}^{56} \times \{0,1\}^{64} \to \{0,1\}^{64}$$

DES Construction

function $\mathrm{DES}_K(M)$ // $|K| = 56$ and $|M| = 64$

$(K_1, \ldots, K_{16}) \leftarrow \mathit{KeySchedule}(K)$ // $|K_i| = 48$ for $1 \leq i \leq 16$

$M \leftarrow \mathit{IP}(M)$

Parse M as $L_0 \,\|\, R_0$ // $|L_0| = |R_0| = 32$

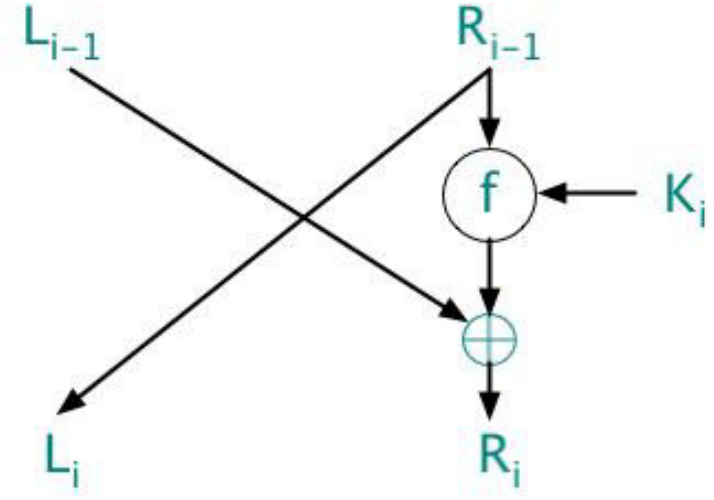
for $i = 1$ to 16 do

$L_i \leftarrow R_{i-1}$; $R_i \leftarrow f(K_i, R_{i-1}) \oplus L_{i-1}$

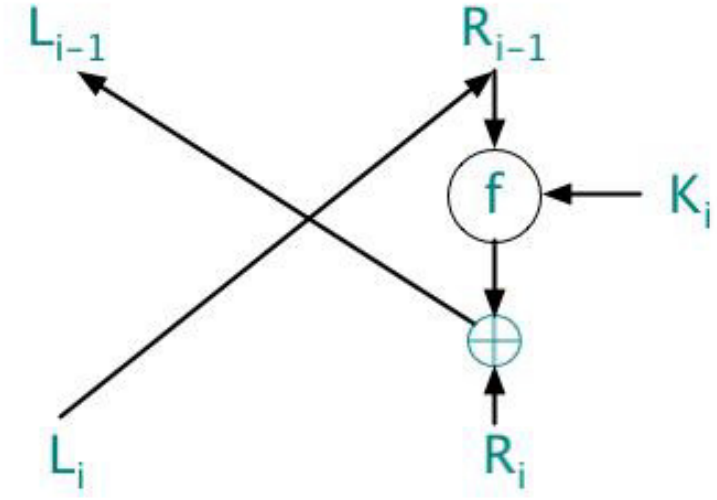
$C \leftarrow \mathit{IP}^{-1}(L_{16} \,\|\, R_{16})$

return C

Round i:



Invertible given K_i:



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$(K_1, \ldots, K_{16}) \leftarrow \mathit{KeySchedule}(K)$ // $|K_i| = 48$ for $1 \leq i \leq 16$

$C \leftarrow \mathit{IP}(C)$

Parse C as $L_{16} \,\|\, R_{16}$

for $i = 16$ downto 1 do

$R_{i-1} \leftarrow L_i$; $L_{i-1} \leftarrow f(K_i, R_{i-1}) \oplus R_i$

$M \leftarrow \mathit{IP}^{-1}(L_0 \,\|\, R_0)$

return M

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IP

58	50	42	34	26	18	10	2
60	52	44	36	28	20	12	4
62	54	46	38	30	22	14	6
64	56	48	40	32	24	16	8
57	49	41	33	25	17	9	1
59	51	43	35	27	19	11	3
61	53	45	37	29	21	13	5
63	55	47	39	31	23	15	7

IP^{-1}

40	8	48	16	56	24	64	32
39	7	47	15	55	23	63	31
38	6	46	14	54	22	62	30
37	5	45	13	53	21	61	29
36	4	44	12	52	20	60	28
35	3	43	11	51	19	59	27
34	2	42	10	50	18	58	26
33	1	41	9	49	17	57	25

DES Construction

function $f(J, R)$ // $|J| = 48$ and $|R| = 32$

$R \leftarrow E(R)$; $R \leftarrow R \oplus J$

Parse R as $R_1 \| R_2 \| R_3 \| R_4 \| R_5 \| R_6 \| R_7 \| R_8$ // $|R_i| = 6$ for $1 \leq i \leq$

for $i = 1, \ldots, 8$ do

$R_i \leftarrow \mathbf{S}_i(R_i)$ // Each S-box returns 4 bits

$R \leftarrow R_1 \| R_2 \| R_3 \| R_4 \| R_5 \| R_6 \| R_7 \| R_8$ // $|R| = 32$ bits

$R \leftarrow P(R)$; return R

E

32	1	2	3	4	5
4	5	6	7	8	9
8	9	10	11	12	13
12	13	14	15	16	17
16	17	18	19	20	21
20	21	22	23	24	25
24	25	26	27	28	29
28	29	30	31	32	1

P

16	7	20	21
29	12	28	17
1	15	23	26
5	18	31	10
2	8	24	14
32	27	3	9
19	13	30	6
22	11	4	25

S-boxes

$\mathbf{S}_1$:

		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	14	4	13	1	2	15	11	8	3	10	6	12	5	9	0	7
0	1	0	15	7	4	14	2	13	1	10	6	12	11	9	5	3	8
1	0	4	1	14	8	13	6	2	11	15	12	9	7	3	10	5	0
1	1	15	12	8	2	4	9	1	7	5	11	3	14	10	0	6	13

$\mathbf{S}_2$:

		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	15	1	8	14	6	11	3	4	9	7	2	13	12	0	5	10
0	1	3	13	4	7	15	2	8	14	12	0	1	10	6	9	11	5
1	0	0	14	7	11	10	4	13	1	5	8	12	6	9	3	2	15
1	1	13	8	10	1	3	15	4	2	11	6	7	12	0	5	14	9

$\mathbf{S}_3$:

		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	10	0	9	14	6	3	15	5	1	13	12	7	11	4	2	8
0	1	13	7	0	9	3	4	6	10	2	8	5	14	12	11	15	1
1	0	13	6	4	9	8	15	3	0	11	1	2	12	5	10	14	7
1	1	1	10	13	0	6	9	8	7	4	15	14	3	11	5	2	12

Key Recovery Attacks on Function Families

Let E: Keys $\times$ Dom $\to$ Rng be a family of functions. It is known to the adversary A.

Def: We say that $K' \in$ Keys is *consistent* with $(M_1, C_1), \ldots, (M_q, C_q)$ if $E(K', M_i) = C_i$ for all $1 \leq i \leq q$.

Key-recovery security game, informally:

- A *target key* $K \xleftarrow{\$}$ Keys is selected but not given to A.
- A can submit a plaintext $M \in$ Dom and get back $C = E(K, M)$, in this way gathering input-output examples $(M_1, C_1), \ldots, (M_q, C_q)$ of E_K.
- A outputs a "guess" K'
- A wins if K' is consistent with $(M_1, C_1), \ldots, (M_q, C_q)$.

Key Recovery Attacks on Function Families

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- A outputs a "guess" K'
- A wins if K' is consistent with $(M_1, C_1), \ldots, (M_q, C_q)$.

For many block ciphers, if K' is consistent with K, then $K' = K$, so the attack recovers the target key.

About the model: Certainly A should be given $C_1, \ldots, C_q$. But why does A get to pick $M_1, \ldots, M_q$? Reasons include a posteriori revelation of data, a priori knowledge of context, and just being conservative!

Key recovery game and advantage

Let E: Keys $\times$ Dom $\to$ Rng be a family of functions, and A an adversary.

Game KR_E

procedure Initialize
$K \xleftarrow{\$}$ Keys; $i \leftarrow 0$

procedure Fn(M)
$i \leftarrow i + 1$; $M_i \leftarrow M$
$C_i \leftarrow E(K, M_i)$
Return C_i

procedure Finalize(K')
win $\leftarrow$ true
For $j = 1, \ldots, i$ do
    If $E(K', M_j) \neq C_j$ then win $\leftarrow$ false
    If $M_j \in \{M_1, \ldots, M_{j-1}\}$ then win $\leftarrow$ false
Return win

$$\mathbf{Adv}^{\mathrm{kr}}_E(A) = \Pr[\mathrm{KR}^A_E \Rightarrow \mathrm{true}]$$

We say that A is a q-query adversary if it makes q (distinct) queries to its **Fn** oracle.

Running a game with an adversary

- First **Initialize** executes
- Now A can call (query) **Fn** on any input M of its choice. It can make as many queries as it wants
- Eventually A will halt with an output K' which is automatically viewed as the input to **Finalize**
- The game returns whatever **Finalize** returns
- The advantage of A is the probability that the game returns true

$\mathbf{Adv}^{\mathrm{kr}}_{E}(A)$ will depend on the number q of queries that A makes and its running time.

Exhaustive Key Search attack

Let E: Keys $\times$ Dom $\to$ Rng be a function family with Keys $= \{T_1, \ldots, T_N\}$ and Dom $= \{x_1, \ldots, x_d\}$. Let $1 \leq q \leq d$ be a parameter.

adversary A_{eks}

For $j = 1, \ldots, q$ do $M_j \leftarrow x_j$; $C_j \leftarrow \mathbf{Fn}(M_j)$
For $i = 1, \ldots, N$ do
    if $(\forall j \in \{1, \ldots, q\} \ : \ E(T_i, M_j) = C_j)$ then return T_i

Then $\mathbf{Adv}^{\text{kr}}_E(A_{\text{eks}}) = 1$ because $K \in \{T_1, \ldots, T_N\}$ and K is consistent with $(M_1, C_1), \ldots, (M_q, C_q)$.

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Then $\mathbf{Adv}^{\text{kr}}_E(A_{\text{eks}}) = 1$ because $K \in \{T_1, \ldots, T_N\}$ and K is consistent with $(M_1, C_1), \ldots, (M_q, C_q)$.

Informal claim: Think of q as small, like $q \in \{1, 2, 3\}$. Suppose E: $\{0,1\}^k \times \{0,1\}^\ell \to \{0,1\}^\ell$ is a standard block cipher. Then as long as $q > k/\ell$, empirical evidence says that the attack returns the target key K itself.

Exercise: Target key recovery

Let E: Keys $\times$ Dom $\to$ Rng be a family of functions and A an adversary. The following measures A's ability to find the target key:

Game TKR_E	**procedure Fn**(M)
procedure Initialize	Return $E(K, M)$
$K \xleftarrow{\$}$ Keys	**procedure Finalize**(K')
	Return $(K = K')$

Let $\mathbf{Adv}^{\mathrm{tkr}}_E(A) = \Pr[\mathrm{TKR}^A_E \Rightarrow \text{true}]$.

Let $k, \ell \geq 1$ be given integers. Present in pseudocode a blockcipher E: $\{0,1\}^k \times \{0,1\}^\ell \to \{0,1\}^\ell$ for which you do the following:

(1) Given any integer $q \leq 2^\ell$, present in pseudocode a q-query adversary A_q with $\mathbf{Adv}^{\mathrm{kr}}_E(A_q) = 1$.

(2) Prove that $\mathbf{Adv}^{\mathrm{tkr}}_E(A) \leq 2^{-k}$ for any adversary A.

How long does exhaustive key search take?

DES can be computed at 1.6 Gbits/sec in hardware.

DES plaintext $=$ 64 bits

Chip can perform $(1.6 \times 10^9)/64 = 2.5 \times 10^7$ DES computations per second

Expect A_{eks} $(q = 1)$ to succeed in 2^{55} DES computations, so it takes time

$$\begin{aligned} \frac{2^{55}}{2.5 \times 10^7} &\approx 1.4 \times 10^9 \text{ seconds} \\ &\approx 45 \text{ years!} \end{aligned}$$

Key Complementation $\Rightarrow$ 22.5 years

But this is prohibitive. Does this mean DES is secure?

Differential and linear cryptanalysis

Exhaustive key search is a generic attack: Did not attempt to "look inside" DES and find/exploit weaknesses.

The following non-generic key-recovery attacks on DES have advantage close to one and running time smaller than 2^{56} DES computations:

Attack	when	q, running time
Differential cryptanalysis	1992	2^{47}
Linear cryptanalysis	1993	2^{44}

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Attack	when	q, running time
Differential cryptanalysis	1992	2^{47}
Linear cryptanalysis	1993	2^{44}

But merely storing 2^{44} input-output pairs requires 281 Tera-bytes.

In practice these attacks were prohibitively expensive.

EKS revisited

adversary A_{eks}

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Observation: The E computations can be performed in parallel!

EKS revisited

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Observation: The E computations can be performed in parallel!

In 1993, Wiener designed a dedicated DES-cracking machine:

- $1 million
- 57 chips, each with many, many DES processors
- Finds key in 3.5 hours

RSA DES challenges

$K \xleftarrow{\$} \{0,1\}^{56}$; $Y \leftarrow \mathrm{DES}(K, X)$; Publish Y on website.
Reward for recovering X

Challenge	Post Date	Reward	Result
I	1997	\$10,000	Distributed.Net: 4 months
II	1998	Depends how fast you find key	Distributed.Net: 41 days. EFF: 56 hours
III	1998	As above	$<$ 28 hours

DES security summary

DES is considered broken because its short key size permits rapid key-search.

But DES is a very strong design as evidenced by the fact that there are no practical attacks that exploit its structure.

2DES

Block cipher $2DES : \{0,1\}^{112} \times \{0,1\}^{64} \to \{0,1\}^{64}$ is defined by

$$2DES_{K_1 K_2}(M) = DES_{K_2}(DES_{K_1}(M))$$

- Exhaustive key search takes 2^{112} DES computations, which is too much even for machines
- Resistant to differential and linear cryptanalysis.

Meet-in-the-middle attack on 2DES

Suppose $K_1 K_2$ is a target 2DES key and adversary has M, C such that

$$C = 2DES_{K_1 K_2}(M) = DES_{K_2}(DES_{K_1}(M))$$

Then

$$DES_{K_2}^{-1}(C) = DES_{K_1}(M)$$

Meet-in-the-middle attack on 2DES

Suppose $DES^{-1}_{K_2}(C) = DES_{K_1}(M)$ and $T_1, \ldots, T_N$ are all possible DES keys, where $N = 2^{56}$.

T_1	$DES(T_1, M)$
T_i	$DES(T_i, M)$
T_N	$DES(T_N, M)$

Table L

$DES^{-1}(T_1, C)$	T_1
$DES^{-1}(T_j, C)$	T_j
$DES^{-1}(T_N, C)$	T_N

Table R

Attack idea:

- Build L,R tables

Meet-in-the-middle attack on 2DES

Suppose $DES^{-1}_{K_2}(C) = DES_{K_1}(M)$ and $T_1, \ldots, T_N$ are all possible DES keys, where $N = 2^{56}$.

$K_1 \rightarrow$

T_1	$DES(T_1, M)$
T_i	$DES(T_i, M)$
T_N	$DES(T_N, M)$

Table L

$\overset{\text{equal}}{\longleftrightarrow}$

$DES^{-1}(T_1, C)$	T_1
$DES^{-1}(T_j, C)$	T_j
$DES^{-1}(T_N, C)$	T_N

Table R

$\leftarrow K_2$

Attack idea:

- Build L,R tables
- Find i, j s.t. $L[i] = R[j]$
- Guess that $K_1 K_2 = T_i T_j$

Meet-in-the-middle attack on 2DES

Let $T_1, \ldots, T_{2^{56}}$ denote an enumeration of DES keys.

adversary A_{MinM}

$M_1 \leftarrow 0^{64}$; $C_1 \leftarrow \mathbf{Fn}(M_1)$
for $i = 1, \ldots, 2^{56}$ do $L[i] \leftarrow \mathrm{DES}(T_i, M_1)$
for $j = 1, \ldots, 2^{56}$ do $R[j] \leftarrow \mathrm{DES}^{-1}(T_j, C_1)$
$S \leftarrow \{ (i,j) : L[i] = R[j] \}$
Pick some $(l, r) \in S$ and return $T_l \,\|\, T_r$

Attack takes about 2^{57} DES/DES^{-1} computations and has $\mathbf{Adv}^{\mathrm{kr}}_{\mathrm{2DES}}(A_{\mathrm{MinM}}) = 1$.

This uses $q = 1$ and is unlikely to return the target key. For that one should extend the attack to a larger value of q.

3DES

Block ciphers

$$\mathrm{3DES3} : \{0,1\}^{168} \times \{0,1\}^{64} \to \{0,1\}^{64}$$

$$\mathrm{3DES2} : \{0,1\}^{112} \times \{0,1\}^{64} \to \{0,1\}^{64}$$

are defined by

$$\mathrm{3DES3}_{K_1 \,\|\, K_2 \,\|\, K_3}(M) \;=\; \mathrm{DES}_{K_3}(\mathrm{DES}^{-1}_{K_2}(\mathrm{DES}_{K_1}(M))$$

$$\mathrm{3DES2}_{K_1 \,\|\, K_2}(M) \;=\; \mathrm{DES}_{K_2}(\mathrm{DES}^{-1}_{K_1}(\mathrm{DES}_{K_2}(M))$$

Meet-in-the-middle attack on 3DES3 reduces its "effective" key length to 112.

Block size limitation

Later we will see "birthday" attacks that "break" a block cipher $E : \{0,1\}^k \times \{0,1\}^\ell \to \{0,1\}^\ell$ in time $2^{\ell/2}$

For DES this is $2^{64/2} = 2^{32}$ which is small, and this is unchanged for 2DES and 3DES.

Would like a larger block size.

AES

1998: NIST announces competition for a new block cipher

- key length 128
- block length 128
- faster than DES in software

Submissions from all over the world: MARS, Rijndael, Two-Fish, RC6, Serpent, Loki97, Cast-256, Frog, DFC, Magenta, E2, Crypton, HPC, Safer+, Deal

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2001: NIST selects Rijndael to be AES.

AES

function $\mathsf{AES}_K(M)$
    $(K_0, \ldots, K_{10}) \leftarrow \mathit{expand}(K)$
    $s \leftarrow M \oplus K_0$
    for $r = 1$ to 10 do
        $s \leftarrow S(s)$
        $s \leftarrow \mathit{shift\text{-}rows}(s)$
        if $r \leq 9$ then $s \leftarrow \mathit{mix\text{-}cols}(s)$ fi
        $s \leftarrow s \oplus K_r$
    end for
    return s

- Fewer tables than DES
- Finite field operations

The AES movie

http://www.youtube.com/watch?v=H2LlHOw_ANg

Implementing AES

	Code size	**Performance**
Pre-compute and store round function tables	largest	fastest
Pre-compute and store S-boxes only	smaller	slower
No pre-computation	smallest	slowest

AES-NI: Hardware for AES, now present on most processors. Your laptop may have it! Can run AES at around 1 cycle/byte. VERY fast!

Security of AES

Best known key-recovery attack [BoKhRe11] takes $2^{126.1}$ time, which is only marginally better than the 2^{128} time of EKS.

There are attacks on reduced-round versions of AES as well as on its sibling algorithms AES192, AES256. Many of these are "related-key" attacks. There are also effective side-channel attacks on AES such as "cache-timing" attacks [Be05,OsShTr05].

Exercise

Define $F\colon\ \{0,1\}^{256} \times \{0,1\}^{256} \to \{0,1\}^{256}$ by

Alg $F_{K_1 \| K_2}(x_1 \| x_2)$

$y_1 \leftarrow \mathrm{AES}^{-1}(K_1, x_1 \oplus x_2)$; $y_2 \leftarrow \mathrm{AES}(K_2, \overline{x_2})$
Return $y_1 \| y_2$

for all 128-bit strings K_1, K_2, x_1, x_2, where $\overline{x}$ denotes the bitwise complement of x. (For example $\overline{01} = 10$.) Let T_{AES} denote the time for one computation of AES or AES^{-1}. Below, running times are worst-case and should be functions of T_{AES}.

Exercise

1. Prove that F is a blockcipher.

2. What is the running time of a 4-query exhaustive key-search attack on F?

3. Give a 4-query key-recovery attack in the form of an adversary A specified in pseudocode, achieving $\mathbf{Adv}^{\mathrm{kr}}_{F}(A) = 1$ and having running time $\mathcal{O}(2^{128} \cdot T_{\mathsf{AES}})$ where the big-oh hides some small constant.

Limitations of security against key recovery

So far, a block cipher has been viewed as secure if it resists key recovery, meaning there is no efficient adversary A having $\mathbf{Adv}^{\mathrm{kr}}_{E}(A) \approx 1$.

Is security against key recovery enough?

Not really. For example define E: $\{0,1\}^{128} \times \{0,1\}^{256} \to \{0,1\}^{256}$ by

$$E_K(M[1]M[2]) = M[1] \| \mathrm{AES}_K(M[2])$$

This is as secure against key-recovery as AES, but not a "good" blockcipher because half the message is in the clear in the ciphertext.

So what?

Possible reaction: But DES, AES are not designed like E above, so why does this matter?

Answer: It tells us that security against key recovery is not, as a block-cipher property, sufficient for security of uses of the block cipher.

As designers and users we want to know what properties of a block cipher give us security when the block cipher is used.

So what is a "good" block cipher?

Possible Properties	Necessary?	Sufficient?
security against key recovery	YES	NO!
hard to find M given $C = E_K(M)$	YES	NO!
$\vdots$		

We can't define or understand security well via some such (indeterminable) list.

We want a single "master" property of a block cipher that is sufficient to ensure security of common usages of the block cipher.