Problem Set 2

Due: Wednesday January 19, 2011, in class.

The Computability Crib Sheet, available from the course notes section of the class web page, is useful background reading. In all problems the languages are over the alphabet $\Sigma = \{0, 1\}$. For Problem 1, use reductions from the blank tape halting problem or its complement. The examples in Section 4 of the Computability Crib Sheet can be used as models in this regard.

Problem 1. [40 points] Prove that the following language is neither r.e. nor co-r.e.:

$$L = \{ \langle M_1, M_2 \rangle : M_1, M_2 \text{ are TMs and } M_1(\varepsilon) \text{ halts and } M_2(\varepsilon) \text{ loops} \}.$$ 

Problem 2. [30 points] Let $A, B, D$ be languages. We say that $D$ separates $A$ from $B$ if $A \subseteq D$ and $D \cap B = \emptyset$. We say that $A, B$ are separable there exists a decidable language $D$ such that $D$ separates $A$ from $B$. Now let

$$A = \{ \langle M \rangle : M(\langle M \rangle) \text{ rejects} \}$$

$$B = \{ \langle M \rangle : M(\langle M \rangle) \text{ accepts} \}.$$ 

Show that $A, B$ are not separable.

Hint: Assume to the contrary that they are separable by decidable language $D$ and derive a contradiction (thereby showing $D$ could not exist after all) using ideas similar to those in the proof of undecidability of the halting problem.