Problem Set 1

Due: Wednesday January 12, 2011, in class.

The Computability Crib Sheet, available from the course notes section of the class web page, is useful background reading. In all problems the languages are over the alphabet $\Sigma = \{0, 1\}$.

Problem 1. [40 points] If $f: \Sigma^* \rightarrow \Sigma^*$ is a function we define the image of $f$ as

$$\text{Img}(f) = \{ f(w) : w \in \Sigma^* \}.$$ 

Let $L$ be a non-empty language. Prove that $L$ is r.e. if and only if there exists a computable function $f: \Sigma^* \rightarrow \Sigma^*$ such that $L = \text{Img}(f)$.

Problem 2. [30 points] Prove that the following language is decidable:

$$A = \{ \langle M \rangle : \text{The head of TM } M \text{ stays within the first 2011 tape squares in } M's \text{ computation on input } \varepsilon \}.$$