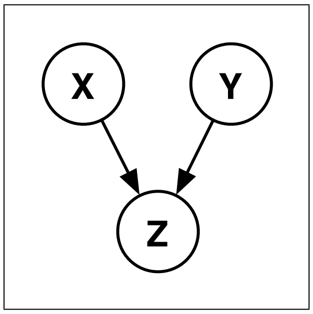
CSE 150. Homework 3 *Summer 2014*

Out: *Tue Aug 12*
Due: *Fri Aug 15*

3.1 Noisy-OR



Nodes: $X \in \{0,1\}$, $Y \in \{0,1\}$, $Z \in \{0,1\}$

Noisy-OR CPT: $P(Z=1|X,Y) = 1 - (1-p_x)^X\,(1-p_y)^Y$

Parameters: $p_x \in [0,1]$, $p_y \in [0,1]$, $p_y > p_x$

Suppose that the nodes in this network represent binary random variables and that the CPT for $P(Z|X,Y)$ is parameterized by a noisy-OR model, as shown above. Suppose also that

$$0 < P(X=1) < 1,$$
$$0 < P(Y=1) < 1,$$

while the parameters of the noisy-OR model satisfy:

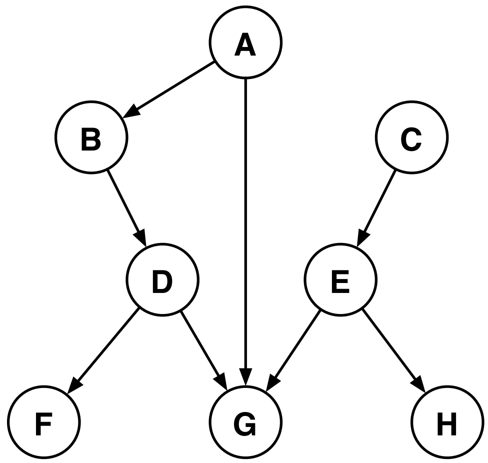
$$0 < p_x < p_y < 1.$$

Consider the following pairs of probabilities. In each case, indicate whether the probability on the left is equal $(=)$, greater than $(>)$, or less than $(<)$ the probability on the right.

(a)	$P(Z=1\|X=0,Y=0)$	☐	$P(Z=1\|X=0,Y=1)$
(b)	$P(Z=1\|X=0,Y=1)$	☐	$P(Z=1\|X=1,Y=0)$
(c)	$P(Z=1\|X=1,Y=0)$	☐	$P(Z=1\|X=1,Y=1)$
(d)	$P(X=1)$	☐	$P(X=1\|Y=1)$
(e)	$P(X=1)$	☐	$P(X=1\|Z=1)$
(f)	$P(X=1\|Y=1,Z=1)$	☐	$P(X=1\|Z=1)$
(g)	$P(X=1,Y=1,Z=1)$	☐	$P(X=1)\,P(Y=1)\,P(Z=1)$

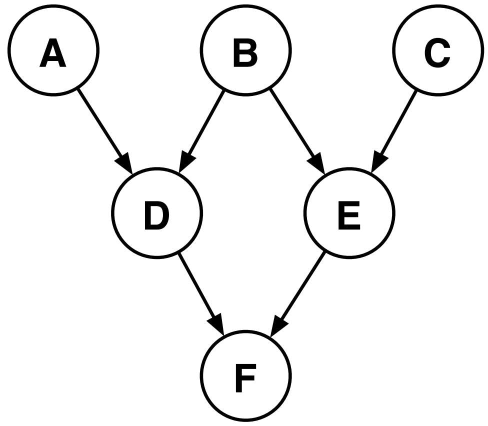
3.2 Conditional independence

For the belief network shown below, indicate whether the following statements of conditional independence are **true (T)** or **false (F)**.



__________	$P(B\|G,C)$	=	$P(B\|G)$
__________	$P(F,G\|D)$	=	$P(F\|D)\,P(G\|D)$
__________	$P(D\|B,F,G)$	=	$P(D\|B,F,G,A)$
__________	$P(A,C)$	=	$P(A)\,P(C)$
__________	$P(F,H)$	=	$P(F)\,P(H)$
__________	$P(F,C\|G)$	=	$P(F\|G)\,P(C\|G)$
__________	$P(D,E\|F,H)$	=	$P(D\|F)\,P(E\|H)$
__________	$P(D,E,G)$	=	$P(D)\,P(E)\,P(G\|D,E)$
__________	$P(H\|C)$	=	$P(H\|A,B,C,D,F)$
__________	$P(H\|A,C)$	=	$P(H\|A,C,G)$

3.3 Subsets



For the belief network shown above, consider the following statements of conditional independence. Indicate the largest subset of nodes $\mathcal{S} \subset \{A, B, C, D, E, F\}$ for which each statement is true. Note that one possible answer is the empty set $\mathcal{S} = \emptyset$ or $\mathcal{S} = \{\}$ (whichever notation you prefer). The first one is done as an example.

$$P(A) = P(A|\mathcal{S}) \qquad \mathcal{S} = \{B, C, E\}$$

$$P(A|D) = P(A|\mathcal{S}) \qquad \underline{\hspace{6cm}}$$

$$P(A|B, D) = P(A|\mathcal{S}) \qquad \underline{\hspace{6cm}}$$

$$P(B|D, E) = P(B|\mathcal{S}) \qquad \underline{\hspace{6cm}}$$

$$P(D) = P(D|\mathcal{S}) \qquad \underline{\hspace{6cm}}$$

$$P(D|F) = P(D|\mathcal{S}) \qquad \underline{\hspace{6cm}}$$

$$P(D|E, F) = P(D|\mathcal{S}) \qquad \underline{\hspace{6cm}}$$

$$P(D|A, B) = P(D|\mathcal{S}) \qquad \underline{\hspace{6cm}}$$

$$P(F) = P(F|\mathcal{S}) \qquad \underline{\hspace{6cm}}$$

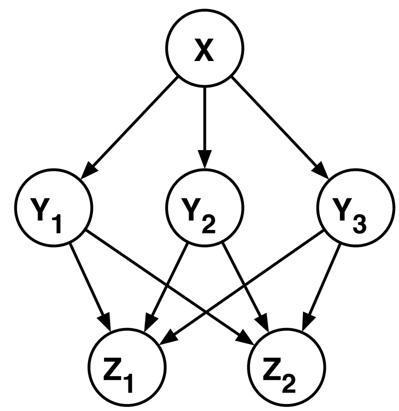
$$P(F|D) = P(F|\mathcal{S}) \qquad \underline{\hspace{6cm}}$$

$$P(F|D, E) = P(F|\mathcal{S}) \qquad \underline{\hspace{6cm}}$$

3.4 Node clustering

Consider the belief network shown below over binary variables X, Y_1, Y_2, Y_3, Z_1, and Z_2. The network can be transformed into a polytree by clustering the nodes Y_1, Y_2, and Y_3 into a single node Y. From the CPTs in the original belief network, fill in the missing elements of the CPTs for the polytree.

X	$P(Y_1 = 1\|X)$	$P(Y_2 = 1\|X)$	$P(Y_3 = 1\|X)$
0	0.9	0.8	0.3
1	0.5	0.6	0.7



Y_1	Y_2	Y_3	$P(Z_1 = 1\|Y_1, Y_2, Y_3)$	$P(Z_2 = 1\|Y_1, Y_2, Y_3)$
0	0	0	0.1	0.8
1	0	0	0.2	0.7
0	1	0	0.3	0.6
0	0	1	0.4	0.5
1	1	0	0.5	0.4
1	0	1	0.6	0.3
0	1	1	0.7	0.2
1	1	1	0.8	0.1

Y_1	Y_2	Y_3	Y	$P(Y\|X=0)$	$P(Y\|X=1)$	$P(Z_1=1\|Y)$	$P(Z_2=1\|Y)$
0	0	0	1				
1	0	0	2				
0	1	0	3				
0	0	1	4				
1	1	0	5				
1	0	1	6				
0	1	1	7				
1	1	1	8