

Robust Near-Isometric Matching via Structured Learning of Graphical Models

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Abstract

In our paper [1], we present a model for matching shapes in images that are related by **near-isometric transformations**. We do so by combining two previous approaches: we use the graphical model of [2], which quickly solves isometric matching problems in 2-d point sets. We use a **structured learning** approach similar to that in [3], which parametrises the matching scores used by linear and quadratic assignment. The result is an **efficient** and **exact** shape-matching algorithm, which uses machine learning to parametrise first, second, and **third-order** features.

Common approaches to the matching problem

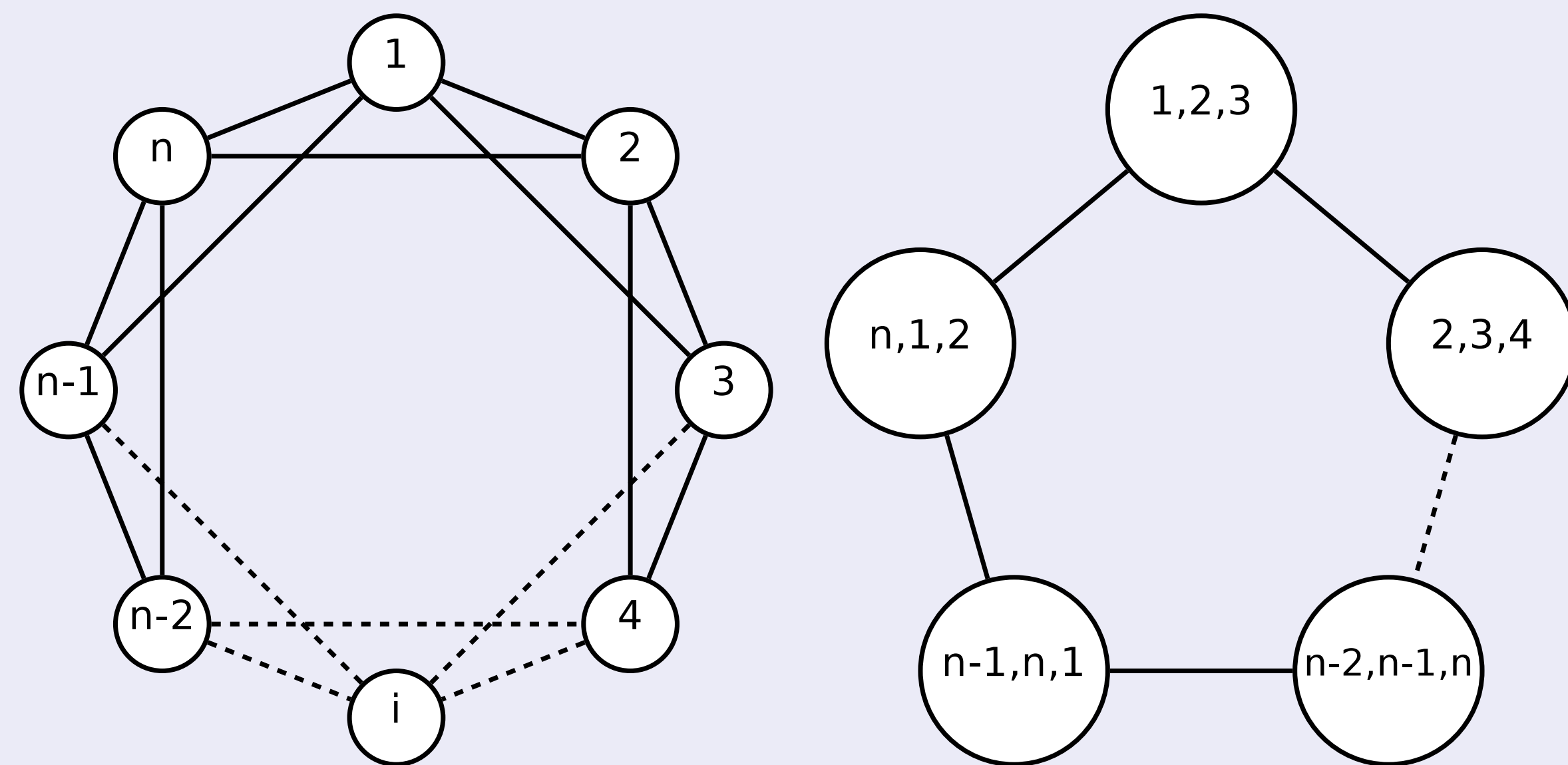
Commonly, the problem of matching objects in images is expressed as a problem of matching graphs, which are attributed by high-dimensional local image descriptors. That is, we want to find a mapping y from nodes in a template graph \mathcal{G} to nodes in a target graph \mathcal{G}' :

$$\operatorname{argmin}_y \sum_{i=1}^{|\mathcal{G}|} \underbrace{\Phi_1(g_i, y(g_i))}_{\text{node features}} + \sum_{i=1}^{|\mathcal{G}|} \sum_{j=1}^{|\mathcal{G}|} \underbrace{\Phi_2(g_i, g_j, y(g_i), y(g_j))}_{\text{edge features}}.$$

Unfortunately, this corresponds to the **quadratic assignment problem**, which is in general **NP-hard**.

Our approach

Instead, we use the recently proposed graphical model from [2], which can solve **near-isometric** matching problems **exactly** in $O(|\mathcal{G}||\mathcal{G}'|^3)$ time:



the graphical model from [2] (left), and its clique-graph (right)

Nodes in this model correspond to points in the template graph, and their assignments correspond to points in the target graph. Because the clique-graph contains only a single loop, **loopy belief-propagation** in this graph will converge to the optimal solution.

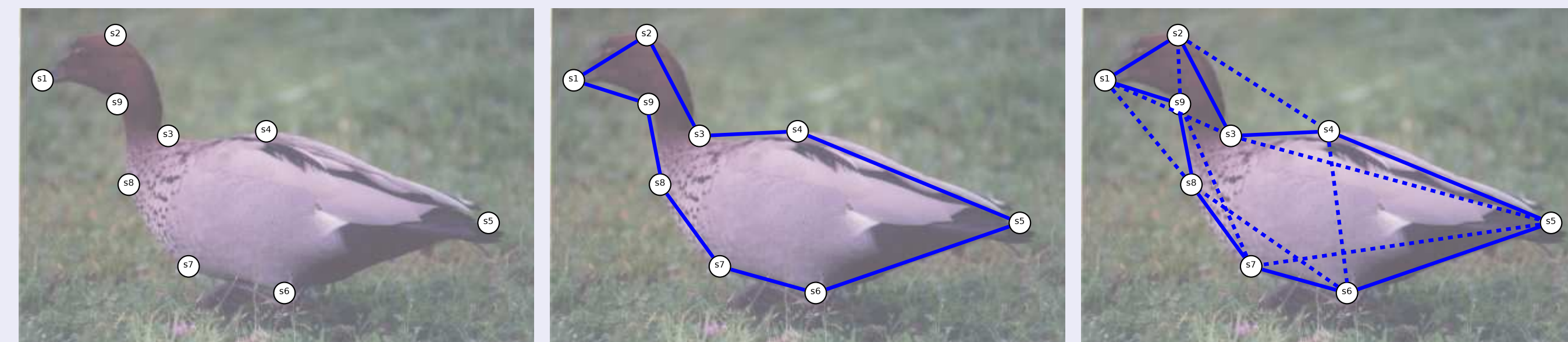
Our energy function

The energy minimisation problem we now want to solve is

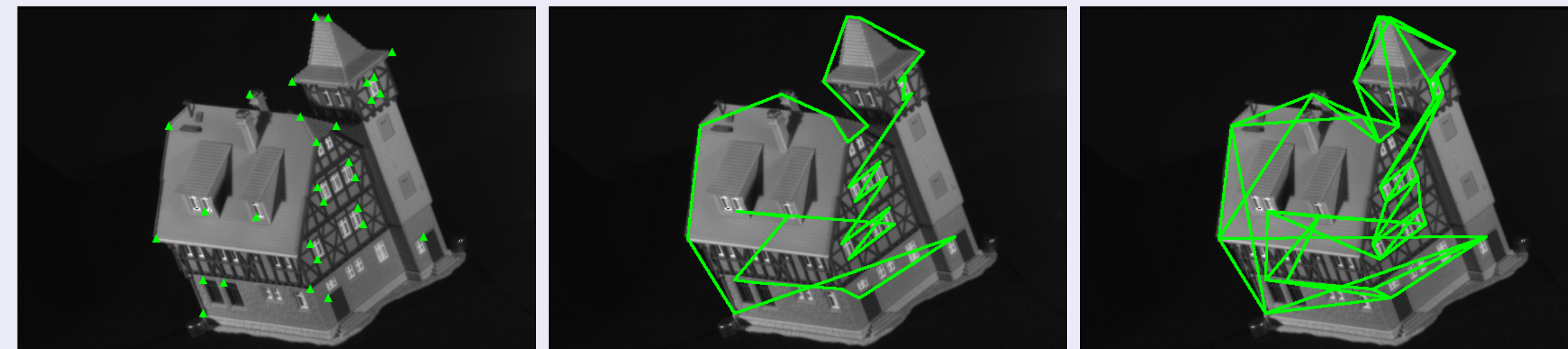
$$\operatorname{argmin}_y \sum_{i=1}^{|\mathcal{G}|} \underbrace{\Phi_3(g_i, g_{i+1}, g_{i+2}, y(g_i), y(g_{i+1}), y(g_{i+2}))}_{\text{third-order features}}.$$

Φ_3 allows us to include first-order features (such as SIFT, Shape Context), second-order features (such as adjacency information, distances), and **third-order** features (such as angles, triangle similarity).

Matching 'shapes' and 'point-patterns'



When the objects being matched are characterised by their shape, our model seems to capture exactly the desired dependencies.

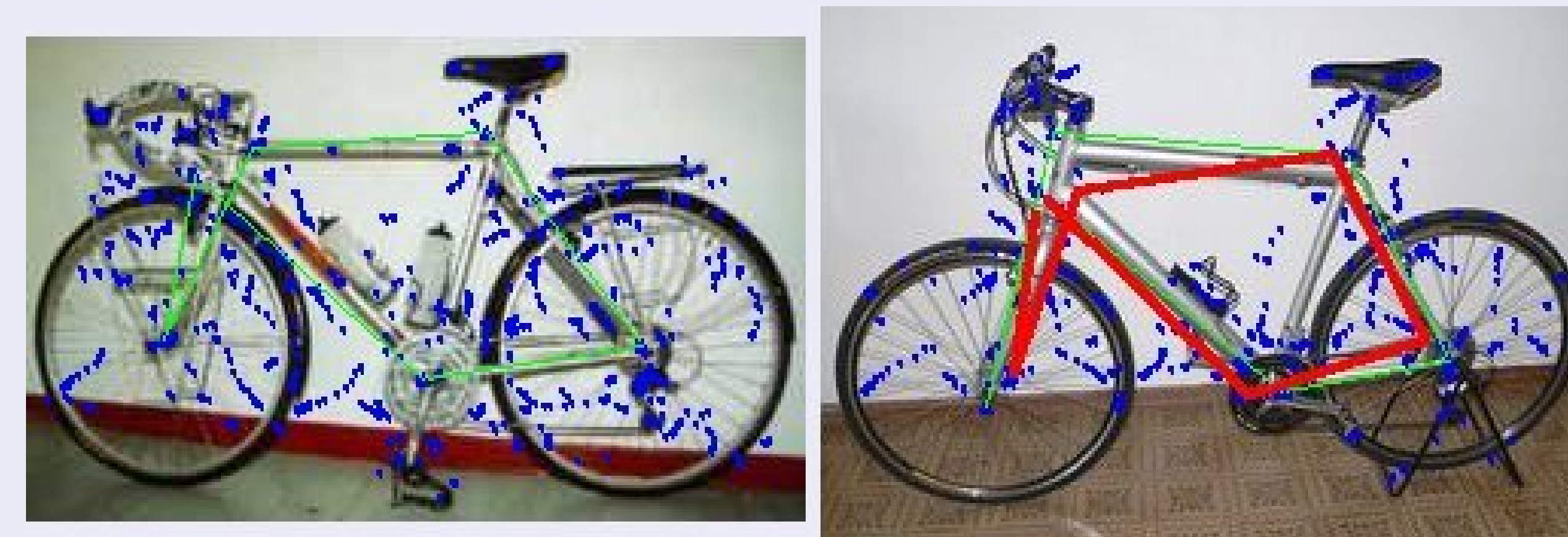


In other cases, we capture only a fraction of the desired dependencies, but we benefit from the addition of third-order features, and from the fact that our model is exact.

Structured learning

Given a collection of labeled training matches, we use a **structured learning** approach similar to that of [3] to parametrise Φ_3 . This allows us to determine the degree to which changes in appearance play a role, versus changes in rotation and scale.

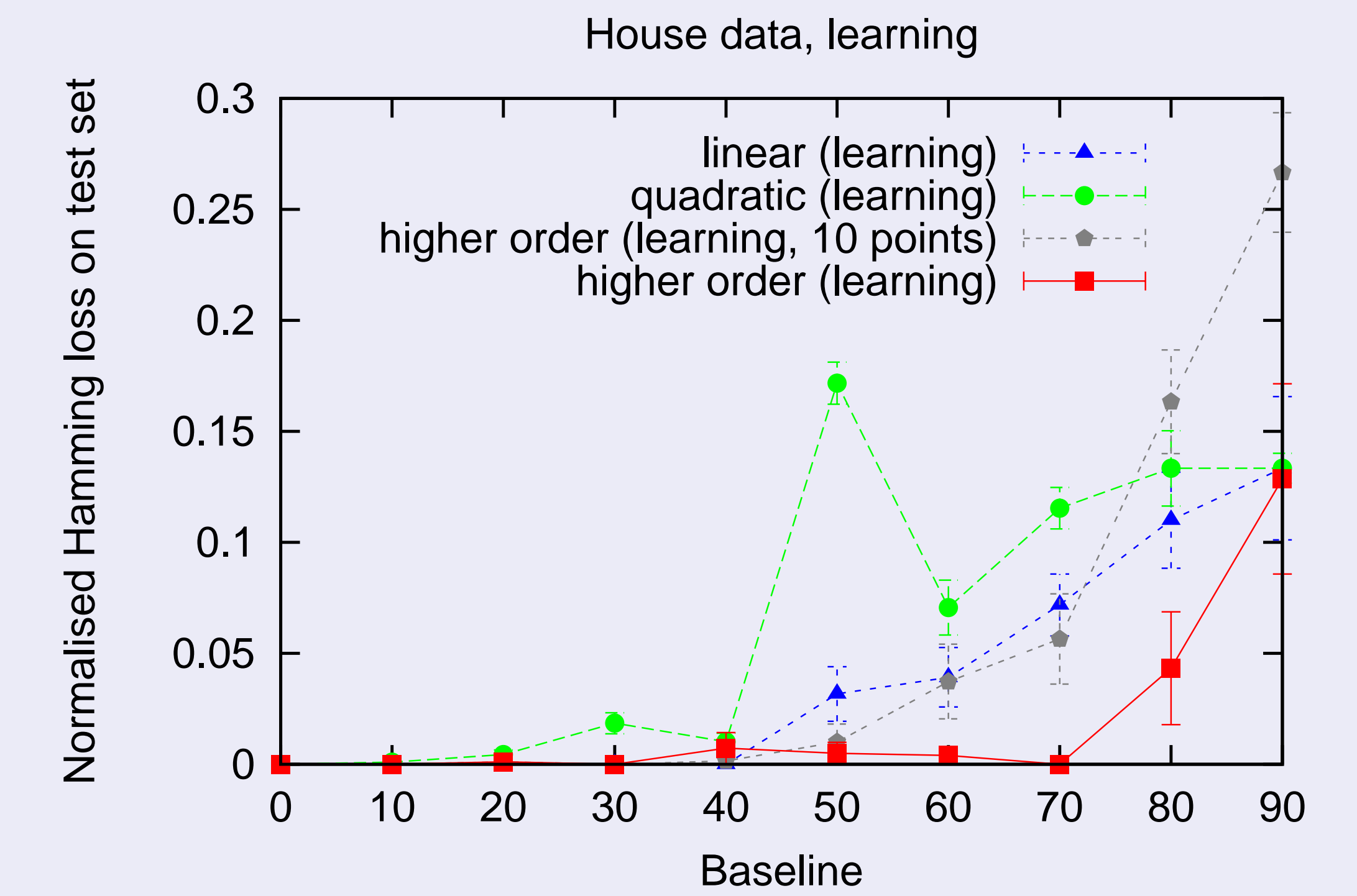
An example match...



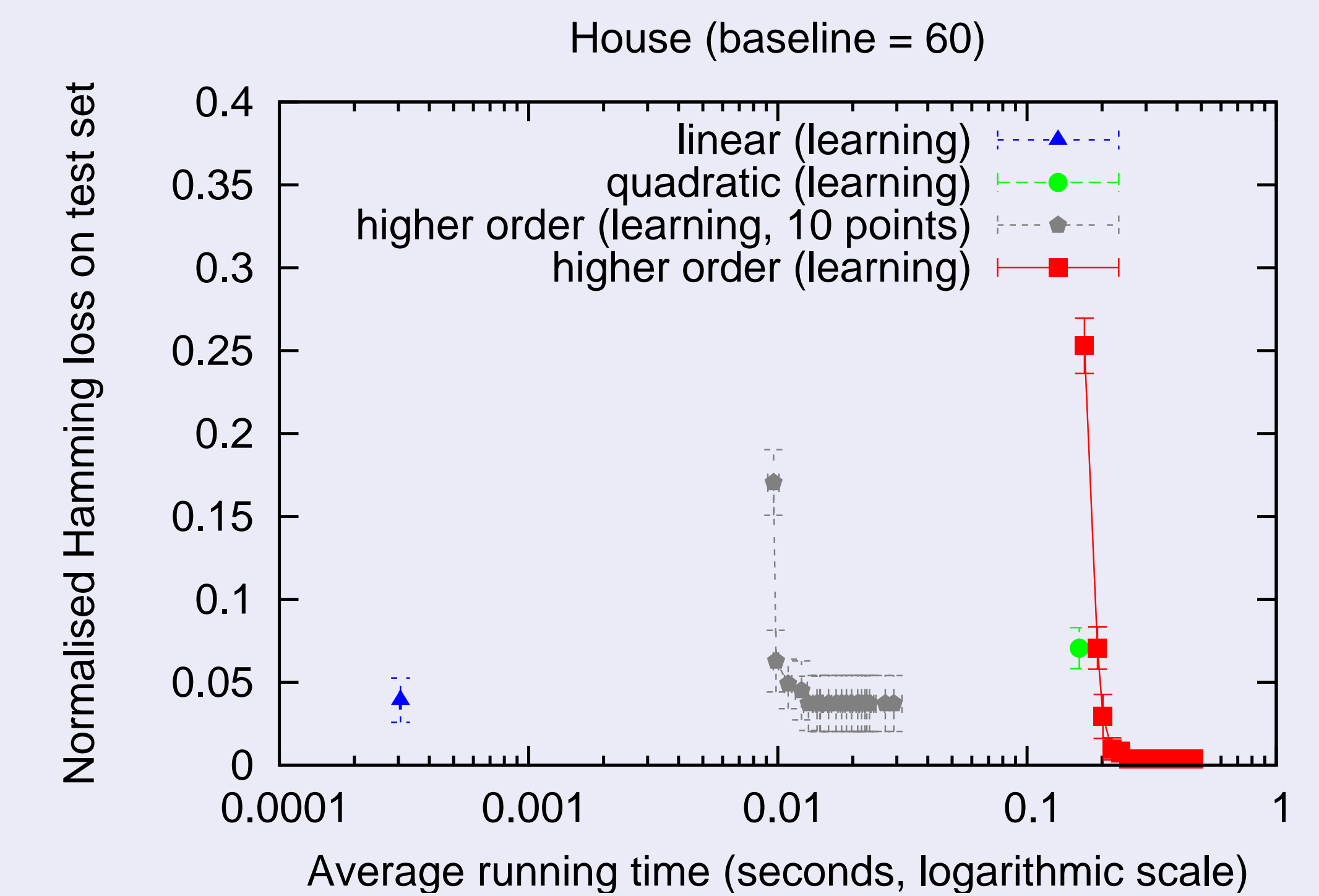
A labeled shape (left), and its match found in another scene. The correct label is shown in green, the inferred label using our model is shown in red.

Results on a video sequence

We compare our method to that of [3] (which parametrises features for linear and quadratic assignment) on frames in a video sequence. The proportion of incorrect labels is reported as the baseline (separation between frames) varies.



By considering only a **'likely subset'** of p points in the target graph, we reduce the running time from $O(|\mathcal{G}||\mathcal{G}'|^3)$ to $O(|\mathcal{G}|p^3)$. With $p = 10$, our algorithm is favourable in terms of performance and running time.



Bibliography

- [1] J. J. McAuley, T. S. Caetano, and A. J. Smola. Robust near-isometric matching via structured learning of graphical models. *NIPS*, 2009.
- [2] J. J. McAuley, T. S. Caetano, and M. S. Barbosa. Graph rigidity, cyclic belief propagation and point pattern matching. *PAMI*, 30(11):2047–2054, 2008.
- [3] T.S. Caetano, L. Cheng, Q.V. Le, and A.J. Smola. Learning graph matching. In *ICCV*, pages 1–8, 2007.