ABSTRACT

We propose an efficient algorithmic framework for time-domain circuit simulation using exponential integrators. This work addresses several critical issues exposed by previous matrix exponential based circuit simulation research, and makes it possible of simulating stiff nonlinear circuit system at a large scale. In this framework, the system’s nonlinearity is treated with exponential Rosenbrock-Euler formulation. The matrix exponential and vector product (MEVP) is computed using invert Krylov subspace method. Our proposed method has several distinguished advantages over conventional formulations (e.g., well-known, Backward Euler with Newton-Raphson method). The Jacobian factorization is performed only for the conductance/resistance matrix $G$, without being performed for the combinations of the capacitance/inductance matrix $C$ and matrix $G$, which are used in traditional implicit formulations. Furthermore, due to the explicit nature of our formulation, we do not need to repeat LU decompositions when adjusting the length of time steps for error controls. Our algorithm is better suited to solving tightly coupled post-layout circuits in the pursuit for full-chip simulation.

BACKGROUND & MOTIVATION

Time-domain nonlinear circuit simulation

$$\frac{dV(t)}{dt} = f(V,t)$$

LOW-ORDER TIME INTEGRATION SCHEME

A well-known BERN: Backward Euler Implicit Formulation

$$C \frac{d^2x_k}{dt^2} + f(x_k,t) = 0$$

Use Newton-Raphson method (NR) to obtain the solution $x_{k+1}$ at (i+1)-th iteration,

$$\Delta t \frac{d^2x_k}{dt^2} = f(x_k,t)$$

The Jacobian matrix is $J(x_k,t)$, where

$$J(x_k,t) = \frac{df(x_k,t)}{dx_k}$$

Once $x_{k+1}$ is updated in the iteration, we need to solve the linear system Eq. (3).

PREVIOUS MATRIX EXPONENTIAL BASED METHOD & MEVP VIA STANDARD KRYLOV SUBSPACE METHOD

$$\text{EXPRESS}_{\text{KRYLOV}}$$

$$\text{EXPRESS}^{\text{KRYLOV}}$$

$$\text{EXPRESS}_{\text{KRYLOV}}$$

HIGH-ORDER INTEGRATION SCHEME USING EXPONENTIAL INTEGRATORS

• ER: Exponential Rosenbrock-Euler formulation in circuit simulation

$$\frac{dV(t)}{dt} = f(V,t)$$

The next time step solution

$$\text{EXPRESS}_{\text{KRYLOV}}$$

Using Newton-Raphson method (NR) to obtain the solution $x_{k+1}$ at (i+1)-th iteration,

$$\Delta t \frac{d^2x_k}{dt^2} = f(x_k,t)$$

The Jacobian matrix is $J(x_k,t)$, where

$$J(x_k,t) = \frac{df(x_k,t)}{dx_k}$$

Once $x_{k+1}$ is updated in the iteration, we need to solve the linear system Eq. (3).

MEVP VIA INVERT KRYLOV SUBSPACE METHOD (MEVP-\text{KS})

• Invert Krylov subspace method

$$K_{\text{EXPRESS}} = \begin{pmatrix} A_1 & B_1 & \cdots & B_{n-1} \\ B_1 & A_2 & \cdots & B_{n-2} \\ \vdots & \vdots & \ddots & \vdots \\ B_{n-1} & B_{n-2} & \cdots & A_n \end{pmatrix}$$

• MEVP:

$$e^{tA} = \text{Block}_{\text{Diag}}(e^{tA_1}, e^{tA_2}, \ldots, e^{tA_n})$$

• Residual-based error estimation for the convergence check in MEVP-\text{KS}

$$r_{\text{EXPRESS}}(t) = \begin{pmatrix} \dot{x}(t) - f(x(t),t) \\ 0 \end{pmatrix}$$

Solve $\text{Gait}(x(t),t) = 0$ and obtain $\dot{x}(t)$:

$$\dot{x}(t) = \text{Gait}(x(t),t)$$

$$b_0 = \dot{x}(t)$$

$$b_1 = \text{Gait}(x(t),t)$$

$$\text{end}$$

$$h_{\text{EXPRESS}} = \frac{1}{m} \sum_{i=1}^{m} |r_{\text{EXPRESS}}(t_i)|$$

The proposed method has several distinguished advantages over conventional formulations (e.g., well-known, Backward Euler with Newton-Raphson method). The Jacobian factorization is performed only for the conductance/resistance matrix $G$, without being performed for the combinations of the capacitance/inductance matrix $C$ and matrix $G$, which are used in traditional implicit formulations. Furthermore, due to the explicit nature of our formulation, we do not need to repeat LU decompositions when adjusting the length of time steps for error controls. Our algorithm is better suited to solving tightly coupled post-layout circuits in the pursuit for full-chip simulation.

EXPERIMENTAL RESULTS

The algorithmic frameworks are implemented in MATLAB and C++. The algorithmic frameworks are implemented in MATLAB and C++. All devices are evaluated in BSIM3 with MATLAB2013a Executable (MEX) external interface, GIC 4.7.3.

Linux workstation

Intel CPU i7 3.4Ghz, 32GB memory.

Utilize single thread mode.

Design & Specifications

• Implementation Timing - Iterations and reduce the number of LU decomposition operations.

Linux workstation

Intel CPU i7 3.4Ghz, 32GB memory.

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Design & Specifications

• Performance (RTs): the runtime of time-domain simulation. SP: the runtime speedup.

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<th>BENK</th>
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<th>RT (SP)</th>
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CONCLUSION

We propose a new and efficient algorithmic framework for time-domain large-scale circuit simulation using exponential integrators:

• By virtue of the stable explicit formulation of solving ODE, we remove Newton-Raphson iterations and reduce the number of LU decomposition operations.

• MEVP is computed efficient invert Krylov subspace method, which also keeps capacitance/inductance matrix $C$ from expensive matrix factorizations, avoids the time-consuming regularization process when $C$ is singular.

• No repeated LU decompositions when adjusting the length of time steps for error controls.

• The proposed method can handle many parasites, strongly coupled and post-layout circuits, while conventional methods are not applicable.

We test our proposed framework (ERER-ER) against standard backward Euler method with Newton-Raphson iterations (BENR). Our framework does not only accelerate the simulation, but also manages to finish the challenging test cases, even when BENR fails.