CORRIGENDA AND COMMENTS ON CHAPTER 2


Correction 2 (Page 12).

\[ T = \prod_{i=0}^{p} T_i \]

should be

\[ T = \sum_{i=0}^{p} T_i \]

Comment 1 (Page 27). Starting with Equation 13 and throughout the remainder of the chapter, it should be noted that in formulas of this type one needs either the characteristic of the field \( R \) to be zero, or that the characteristic of \( R \) not be a divisor of \( |H| \).

Comment 2 (Page 30). In Example 2.5 (c), in the trivial case \( m = 1 \), \( S_e = S_1 \) and so \( S_eS_1 = S_1 \). Thus one must assume \( m \geq 2 \).

Correction 3 (Page 33). In the second line of the equation in Problem 1,

\[ \ldots = \lambda(S^{-1}S)_1 = \lambda \delta_{i1} \]

should be

\[ \ldots = \lambda_1(S^{-1}S)_1 = \lambda_1 \delta_{i1} \]

Correction 4 (Page 52). The lower limit of the sum is missing in the equation:

\[ \varkappa(e_\omega^*) = \frac{1}{|S_m|} \sum_{\sigma \in S_m} \epsilon(\sigma) \prod_{t=1}^{m} \left( \sum_{i=1}^{n} b_{it} f_i(e_{\omega\sigma^{-1}(t)}) \right) \]

It should be:

\[ \varkappa(e_\omega^*) = \frac{1}{|S_m|} \sum_{\sigma \in S_m} \epsilon(\sigma) \prod_{t=1}^{m} \left( \sum_{i=1}^{n} b_{it} f_i(e_{\omega\sigma^{-1}(t)}) \right) \]
Correction 5 (Page 54). In the last line of Equation 29:

\[ = \delta_{\alpha\beta} \sum_{\sigma \in H_\alpha} \chi(\sigma) \]

the subscript \( \alpha \) of \( H_\alpha \) is almost cut off, presumably by the page number.

Correction 6 (Page 59). In the first set of equations in Exercise 9, the product symbol is missing from last line. It should be:

\[ = \sum_{\omega \in G_{n,n}} \frac{1}{\nu(\omega)} \prod_{t=1}^{n} \lambda_{\omega(t)} |\text{per} U[\omega|1,\ldots,n]|^2 \]

Correction 7 (Page 63). In line 10,

\[ (m_V^*)^* \]

should be

\[ (m_V^*)^* \]

Comment 3 (Page 64). The third equality:

\[ = \frac{1}{|H|} \sum_{\sigma \in H} \chi(\sigma) \varphi(f_1,\ldots,f_m)(v_{\sigma^{-1}(1)} \otimes \cdots \otimes v_{\sigma^{-1}(m)}) \]

seems unnecessary, and the next equality seems to follow from the definitions.

Correction 8 (Page 69). In the fourth line of the proof of part (d) of Theorem 4.2, “...are invariant subspaces of \( \bigotimes_1^m T \)...” should be “...are invariant subspaces of \( \bigotimes_1^m V \)...”

Correction 9 (Page 70). In the second line, \( = (S_u^\otimes,S(\bigotimes_1^m T)S_v^\otimes) \) should be \( = (S_u^\otimes,(\bigotimes_1^m T)S_v^\otimes) \).

Correction 10 (Page 78). In the first line,

\[ \text{tr} C_m(A) = \sum_{\alpha \in Q_{m,n}} \prod_{t=1}^{n} \lambda_t^{m_t}(\omega) \]

should be

\[ \text{tr} C_m(A) = \sum_{\alpha \in Q_{m,n}} \prod_{t=1}^{n} \lambda_t^{m_t(\alpha)} \]
Correction 11 (Page 79). In Example 4.4 (f), $A$ needs to be invertible.

Correction 12 (Page 85). In Theorem 4.5 (c), per Exercise 22, the characteristic of $R$ must be zero.

Correction 13 (Page 87). In the proof of Theorem 4.5 (b) in line 9, the upper limit of the sum is incorrect in: $y_i = \sum_{j=1}^{n} c_{ij} x_j$; it should be $y_i = \sum_{j=1}^{m} c_{ij} x_j$.

Correction 14 (Page 89). In the first line, in the equality $z^*_\theta = z^*$ the stars are clipped at the top.

Comment 4 (Page 92). In example 4.6 (d) it is stated “It clearly suffices to prove this for $T_2 > 0$.” Try as I might, I can’t find an easy (or any) proof of this in general. If it happens to be the case that both $T_1$ and $T_2$ are diagonal with respect to the same orthonormal basis, then the result follows by restricting both operators to the subspace spanned by the eigenvectors of $T_2$ corresponding to non-zero eigenvalues. But there’s no reason to suppose that such a basis exists.

Correction 15 (Page 92). In the last line “equivalently $K(T_1) \geq K(T_2) \geq 0$” should be “...$K(T_1) \geq K(T_2) > 0$” according to the assumption discussed in the previous comment.

Correction 16 (Page 94). In line 10, “...since $m < r < n$” should be “...since $m < r \leq n$.”

Correction 17 (Page 95). In lines 2 and 3, $c_1 = \cdots = c_m = c$ and $c_{m+1} = \cdots = c_r = c$ should be, respectively, $c_1 = \cdots = c_{m+1} = c$ and $c_{m+2} = \cdots = c_r = c$.

Correction 18 (Page 99). In Exercise 9, one needs to assume that $\text{char}(R) \neq 2$ in order to use Equation (7) of Section 2.3. In the characteristic 2 case, if $v_i = v_j$, just note that if $\sigma$ is a permutation such that $\sigma(i) = k$ and $\sigma(j) = l$, there is permutation $\sigma'$ such that $\sigma'(i) = l$, $\sigma'(j) = k$ and $\sigma'(p) = \sigma(p)$ otherwise. Then $v_{\sigma(1)} \otimes \cdots \otimes v_{\sigma(m)} = v_{\sigma'(1)} \otimes \cdots \otimes v_{\sigma'(m)}$, and so $v_{\sigma(1)} \otimes \cdots \otimes v_{\sigma(m)} + v_{\sigma'(1)} \otimes \cdots \otimes v_{\sigma(m)} = 0$. Pairing all permutations in $S_m$ this way yields the result.

Comment 5 (Page 100). In Exercise 11, it is stated, “Hence $d_H^X(X) = 0$ whenever $\det(X) = 0$ implies $d_H^X(X)$ is divisible by $\det(X)$.” another statement I can’t easily figure out. Anyway, there’s no need to go through these manipulations of multivariable polynomials. The following proof works as well:

Suppose first that $H$ is a proper subgroup of $S_n$. Then there exists a transposition $(pq) \notin S_n$. Let $X$ be a matrix defined as follows:

\[
X_{kk} = 1, \ k = 1, \ldots, n; \\
X_{pq} = X_{qp} = 1; \\
X_{kl} = 0 \text{ otherwise.}
\]
Since rows $p$ and $q$ of $X$ are equal, $\det(X) = 0$. Now compute $d^H_\chi(X)$. If $k \neq p,q$ and $\sigma(k) \neq k$, $X_{k\sigma(k)} = 0$; hence

$$\prod_{1}^{n} \chi(\sigma)X_{j\sigma(j)} = 0$$

So only permutations $\sigma$ satisfying $\sigma(k) = k$ for $k \neq p,q$ contribute to the value of $d^H_\chi(X)$. But there are only two such permutations: the identity and $(pq)$, and the latter is not in $H$ by hypothesis. So $d^H_\chi(X) = X_{11} \cdots X_{nn} = 1 \neq 0 = \det(X)$. So the conclusion is that if $d^H_\chi(X) = 0$ whenever $X$ is singular, then $H = S_n$. But then by Exercise 13 of Section 2, $\chi \equiv 1$ or $\chi = \epsilon$ (the sign function). In the first case, $d^H_\chi(X) = \text{per}(X)$: if $X$ is the matrix with a 1 in every entry, $X$ is singular but $\text{per}(X) = 1$. So $\chi = \epsilon$ and $d^H_\chi$ is the determinant.

**Correction 19** (Page 103). In Exercise 21, the reader should be asked to, “Show that there exists a non-zero $f \in V^*$ . . .”

**Correction 20** (Page 105). Two corrections to Exercise 26. First, the hypothesis $\rho(T) > m$ is needed to Apply Theorem 4.6. Second, the hint should read, “The transformation $iK(T)$ . . .”

**Comment 6** (Pages 107, 108). In Exercise 31, on Page 107, the decomposition

$$U^*AU = \begin{bmatrix} \lambda_1 & * & \cdots & * \\ 0 & \lambda_2 & * & \cdots \\ \vdots & \vdots & \vdots & \ddots \\ 0 & \cdots & 0 & \lambda_p & * \\ 0 & \cdots & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

is essentially the Schur Decomposition; calling the right side of this equation $T$, the Schur Decomposition of $A$ is $A = UTU^*$. On Page 108, I still haven’t been able to figure out the statement, “Since $SS^*$ is non-singular, it follows that $LL^* = 0$.”