Multiple Choice Questions for Review

1. In each case some information is given about a function. In which case is the information not sufficient to define a function?
   (a) \( f \in \mathbb{R} \), \( 2 \rightarrow 3, \ 1 \rightarrow 4, \ 3 \rightarrow 2 \).
   (b) \( f \in \{>, <, +, \?\} \), \( f = (?, <, +) \).
   (c) \( f \in \mathbb{R} \), \( f = (3,1,2,3) \).
   (d) \( f \in \mathbb{R} \), \( f = (3,1,2,3) \). Domain ordered as follows: >, <, +, ?.
   (e) \( f \in \{>, <, +, \?\} \), \( f = (?, <, +) \). Domain ordered as follows: 3, 2, 1.

2. The following function is in two line form: \( f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 5 & 9 & 2 & 4 & 1 & 3 & 6 & 7 \end{pmatrix} \). Which of the following is a correct cycle form for \( h = f^3 \circ f^{-1} \)?
   (a) \( (1,8,9)(2,3,7,5,6,4) \)
   (b) \( (1,9,8)(2,3,5,7,6,4) \)
   (c) \( (1,9,8)(2,3,7,5,4,6) \)
   (d) \( (1,9,8)(2,3,7,5,6,4) \)
   (e) \( (1,9,8)(3,2,7,5,6,4) \)

3. In each case some information about a function is given to you. Based on this information, which function is an injection?
   (a) \( f \in \mathbb{R} \), \( \text{Coimage}(f) = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}\} \)
   (b) \( f \in \mathbb{R} \), \( \text{Coimage}(f) = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5,6\}\} \)
   (c) \( f \in \mathbb{R} \), \( f^{-1}(2) = \{1,3,5\}, \ f^{-1}(4) = \{2,4\} \)
   (d) \( f \in \mathbb{R} \), \( |\text{Image}(f)| = 4 \)
   (e) \( f \in \mathbb{R} \), \( \text{Coimage}(f) = \{\{1,3,5\}, \{2,4\}\} \)

4. The following function is in two line form: \( f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 5 & 9 & 2 & 4 & 1 & 3 & 6 & 7 \end{pmatrix} \). Which of the following is a correct cycle form for \( h = f^3 \circ f^{-1} \)?
   (a) \( (1,6,8)(2,3,7)(5,6,4) \)
   (b) \( (1,6,8)(2,4,5)(3,7,9) \)
   (c) \( (1,8,6)(2,3,7)(5,9,4) \)
   (d) \( (1,9,8)(2,3,5)(7,6,4) \)
   (e) \( (8,5,9,2,4,1,3,6,7) \)

5. The following permutation is in two line form: \( f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 6 & 4 & 7 & 2 & 9 & 1 & 3 & 5 \end{pmatrix} \). The permutation \( g = (1,2,3) \) is in cycle form. Let \( h = f \circ g \) be the composition of \( f \) and \( g \). Which of the following is a correct cycle form for \( h \)?
Functions

(a) (1, 6, 9, 5, 2, 4, 7)(3, 8)
(b) (1, 8, 3, 4, 7, 2, 6)(5, 9)
(c) (1, 8, 3, 7, 4, 2, 6)(9, 5)
(d) (1, 8, 4, 3, 7, 2, 6)(9, 5)
(e) (8, 6, 4, 7, 9, 1, 2)(3, 5)

6. We want to find the smallest integer \( n > 0 \) such that, for every permutation \( f \) on \( 4 \), the function \( f^n \) is the identity function on \( 4 \). What is the value of \( n \)?
   (a) 4    (b) 6    (c) 12    (d) 24    (e) It is impossible.

7. In the lexicographic list of all strictly decreasing functions in \( 9^5 \), find the successor of 98432.
   (a) 98431    (b) 98435    (c) 98521    (d) 98532    (e) 98543

8. The 16 consecutive points 0, 1, …, 14, 15 have 0 and 15 converted to exterior box boundaries. The interior box boundaries correspond to points 1, 5, 7, 9. This configuration corresponds to
   (a) 9 balls into 5 boxes
   (b) 9 balls into 6 boxes
   (c) 10 balls into 5 boxes
   (d) 10 balls into 6 boxes
   (e) 11 balls into 4 boxes

9. The 16 consecutive points 0, 1, …, 14, 15 have 0 and 15 converted to exterior box boundaries. The interior box boundaries correspond to the strictly increasing functions \( 1 \leq x_1 < x_2 < x_3 < x_4 \leq 14 \) in lex order. How many configurations of balls into boxes come before the configuration \( \bullet | | | | \bullet \bullet \bullet \bullet \bullet \bullet ? \) (Exterior box boundaries are not shown.)
   (a) \( \binom{13}{3} \)    (b) \( \binom{13}{4} \)    (c) \( \binom{14}{3} \)    (d) \( \binom{14}{4} \)    (e) \( \binom{15}{3} \)

10. Suppose \( f \in \mathbb{T}^6 \). How many such functions have \( |\text{Image}(f)| = 4 \)?
    (a) \( S(7, 4) \)    (b) \( S(7, 4)(6) \)    (c) \( S(6, 4)(7) \)    (d) \( S(4, 7)(6) \)    (e) \( S(7, 4)6! \)

11. Let \( X \) be a random variable with distribution \( b(k; n, p) \), \( q = 1 - p \). Let \( Y = (X + 1)^2 \). Then \( E(Y) = ? \)
    (a) \( npq + (np + 1)^2 \)
    (b) \( 2npq + (np + 1)^2 \)
    (c) \( npq + 2(np + 1)^2 \)
    (d) \( (npq)^2 + (np + 1)^2 \)
    (e) \( 2npq(np + 1)^2 \)

12. Let \( X \) and \( Y \) be independent random variables with distribution \( b(k; n, a) \) and \( b(k; n, b) \) respectively. Let \( Z = X + 2Y \). Then, for all \( \epsilon > 0 \), Chebyshev’s inequality guarantees that \( P(\frac{|Z - na - 2nb|}{\epsilon} \geq \epsilon) \) is always less than or equal to what?

Fn-42
Review Questions

(a) \( (na(1-a) + nb(1-b))/\epsilon^2 \)
(b) \( (na(1-a) + 2nb(1-b))/\epsilon^2 \)
(c) \( (na(1-a) + 4nb(1-b))/\epsilon^2 \)
(d) \( (na(1-a) + 2nb(1-b))/\epsilon^3 \)
(e) \( (na(1-a) + 4nb(1-b))/\epsilon^3 \)

13. An 800 page book has 400 misprints. If the misprints are distributed uniformly throughout the book, and the Poisson approximation to the binomial distribution is used to calculate the probability of exactly 2 misprints on page 16, which of the following represents the correct use of the Poisson approximation?

(a) \( e^{-0.5}/8 \)  (b) \( e^{-0.5}/8 \)  (c) \( e^{0.5}/16 \)  (d) \( e^{-0.5}/16 \)  (e) \( e^{-0.5}/32 \)

14. For 40 weeks, once per hour during the 40 hour work week, an employee of Best Cars draws a ball from an urn that contains 1 black and 9 white balls. If black is drawn, a $10 bill is tacked to a bulletin board. At the end of the 40 weeks, the money is given to charity. What is the expected amount of money given?

(a) 1000  (b) 1200  (c) 1400  (d) 1600  (e) 1800

15. For 40 weeks, once per hour during the 40 hour work week, an employee of Best Cars draws a ball from an urn that contains 1 black and 9 white balls. If black is drawn, $10 is tacked to a bulletin board. At the end of the 40 weeks, the money is given to charity. Using the normal approximation, what interval under the standard normal curve should be used to get the area which equals the probability that $1800 or more is given?

(a) from 1.67 to \( \infty \)  (b) from 0 to 1.67  (c) from 0.6 to \( \infty \)  (d) from 0 to 0.6  (e) from 0.6 to 1.67

16. A fair coin is tossed three times. Let \( X \) be the random variable which is one if the first throw is \( T \) (for tails) and the third throw is \( H \) (for heads), zero otherwise. Let \( Y \) denote the random variable that is one if the second and third throws are both \( H \), zero otherwise. The covariance, \( \text{Cov}(X,Y) \) is

(a) \( 1/8 \)  (b) \( -1/8 \)  (c) \( 1/16 \)  (d) \( -1/16 \)  (e) \( 1/32 \)

17. A fair coin is tossed three times. Let \( X \) be the random variable which is one if the first throw is \( T \) (for tails) and the third throw is \( H \) (for heads), zero otherwise. Let \( Y \) denote the random variable that is one if the second and third throws are both \( H \), zero otherwise. The correlation, \( \rho(X,Y) \) is

(a) 0  (b) \( 1/3 \)  (c) \( -1/3 \)  (d) \( 1/8 \)  (e) \( -1/8 \)

18. A fair coin is tossed three times and a \( T \) (for tails) or \( H \) (for heads) is recorded, giving us a 3-long list. Let \( X \) be the random variable which is zero if no \( T \) has another \( T \) adjacent to it, and is one otherwise. Let \( Y \) denote the random variable that counts...
Functions

the number of T's in the three tosses. Let \( h_{X,Y} \) denote the joint distribution of \( X \) and \( Y \). \( h_{X,Y}(1,2) \) equals

(a) \( \frac{5}{8} \)  (b) \( \frac{4}{8} \)  (c) \( \frac{3}{8} \)  (d) \( \frac{2}{8} \)  (e) \( \frac{1}{8} \)

19. Which of the following is equal to \( \text{Cov}(X + Y, X - Y) \), where \( X \) and \( Y \) are random variables on a sample space \( S \)?

(a) \( \text{Var}(X) - \text{Var}(Y) \)
(b) \( \text{Var}(X^2) - \text{Var}(Y^2) \)
(c) \( \text{Var}(X^2) + 2\text{Cov}(X,Y) + \text{Var}(Y^2) \)
(d) \( \text{Var}(X^2) - 2\text{Cov}(X,Y) + \text{Var}(Y^2) \)
(e) \( (\text{Var}(X))^2 - (\text{Var}(Y))^2 \)

20. Which of the following is equal to \( \text{Var}(2X - 3Y) \), where \( X \) and \( Y \) are random variables on \( S \)?

(a) \( 4\text{Var}(X) + 12\text{Cov}(X,Y) + 9\text{Var}(Y) \)
(b) \( 2\text{Var}(X) - 3\text{Var}(Y) \)
(c) \( 2\text{Var}(X) + 6\text{Cov}(X,Y) + 3\text{Var}(Y) \)
(d) \( 4\text{Var}(X) - 12\text{Cov}(X,Y) + 9\text{Var}(Y) \)
(e) \( 2\text{Var}(X) - 6\text{Cov}(X,Y) + 3\text{Var}(Y) \)

21. The strictly decreasing functions in \( 100^3 \) are listed in lex order. How many are there before the function \( (9,5,4) \)?

(a) 18  (b) 23  (c) 65  (d) 98  (e) 180

22. All but one of the following have the same answer. Which one is different?

(a) The number of multisets of size 20 whose elements lie in \( 5 \).
(b) The number of strictly increasing functions from 20 to 24.
(c) The number of subsets of size 20 whose elements lie in \( 24 \).
(d) The number of weakly decreasing 4-lists made from \( 21 \).
(e) The number of strictly decreasing functions from \( 5 \) to \( 24 \).

23. Let \( X \) be a random variable with Poisson distribution \( p(k; \lambda) \) Let \( Y = (X + 2)(X + 1) \). What is the value of \( E(Y) \)?

(a) \( \lambda^2 + 3\lambda + 1 \)
(b) \( \lambda^2 + 3\lambda + 2 \)
(c) \( \lambda^2 + 4\lambda + 2 \)
(d) \( 3\lambda^2 + 3\lambda + 2 \)
(e) \( 4\lambda^2 + 4\lambda + 2 \)

Answers: 1 (c), 2 (d), 3 (a), 4 (b), 5 (a), 6 (c), 7 (c), 8 (c), 9 (a), 10 (c), 11 (a), 12 (c), 13 (b), 14 (d), 15 (a), 16 (c), 17 (b), 18 (d), 19 (a), 20 (d), 21 (c), 22 (e), 23 (c).

Fn-44
Notation Index

\( \exists \) (there exists) \quad Fn-4
\( \forall \) (for all) \quad Fn-4
\( \ni \) (such that) \quad Fn-4
Cov(\( X, Y \)) (covariance) \quad Fn-25
\( \mu_X \) (expectation
or mean) \quad Fn-24
E(\( X \)) (expectation) \quad Fn-24
f \circ g (\) (composition) \quad Fn-7
\( \mathbb{N} \) (first \( n \) integers) \quad Fn-1
\( \mathcal{P}_k(A) \) (\( k \)-subsets of \( A \)) \quad Fn-1
\( S(A) \) (permutations of \( A \)) \quad Fn-7
PER(A) (permutations of \( A \)) \quad Fn-7

Probability notation
\( \mu_X \) (expectation, or
mean) \quad Fn-24
\( \rho(\( X, Y) \) (correlation) \quad Fn-25
\( \sigma_X \) (standard deviation) \quad Fn-25
E(\( X \)) (expectation) \quad Fn-24
Cov(\( X, Y \)) (covariance) \quad Fn-25
Var(\( X \)) (variance) \quad Fn-25
\( \mathbb{Q} \) (rational numbers) \quad Fn-1
\( \mathbb{R} \) (real numbers) \quad Fn-1
\( \rho(\( X, Y) \) (correlation) \quad Fn-25

Set notation
\( \sim A \) (complement) \quad Fn-1
\( A' \) (complement) \quad Fn-1
\( A - B \) (difference) \quad Fn-1
\( A \cap B \) (intersection) \quad Fn-1
\( A \cup B \) (union) \quad Fn-1
\( A \oplus B \) (symmetric
difference) \quad Fn-1
\( A \setminus B \) (difference) \quad Fn-1
\( A \times B \) (Cartesian product) \quad Fn-1
\( A^c \) (complement) \quad Fn-1
\( \mathcal{P}_k(A) \) (\( k \)-subsets of \( A \)) \quad Fn-1
\( \sigma_X \) (standard deviation) \quad Fn-25
Var(\( X \)) (variance) \quad Fn-25
\( \mathbb{Z} \) (integers) \quad Fn-1
Subject Index

Bijection  Fn-3
Binomial distribution  Fn-34
Blocks of a partition  Fn-15

Cartesian product  Fn-1
Central Limit Theorem  Fn-38
Chebyshev’s inequality  Fn-27
Codomain (range) of a function  Fn-2
Coimage of a function  Fn-14
Complement of a set  Fn-1
Composition of functions  Fn-7
Correlation  Fn-25
Covariance  Fn-25
Cycle in a permutation  Fn-9

Decreasing (strictly) function or list  Fn-17
Decreasing (weakly) function or list  Fn-17
Density function  Fn-22
Derangement  Fn-12
Deviation
standard  Fn-25
Direct (Cartesian) product  Fn-1
Distribution  Fn-22
  binomial  Fn-34
  joint  Fn-28
  marginal  Fn-28
  normal  Fn-36
  Poisson  Fn-35
Distribution function
  see Distribution
Domain of a function  Fn-2

Envelope game  Fn-2

Event  Fn-21
  independent pair  Fn-29
Expectation of a random variable  Fn-24

Function
  bijection  Fn-3
  codomain (range) of  Fn-2
  coimage of  Fn-14
  composition of  Fn-7
  density  Fn-22
  distribution, see Distribution
  domain of  Fn-2
  image of  Fn-14
  image of and Stirling numbers (set partitions)  Fn-15
  injective (one-to-one)  Fn-3
  inverse  Fn-3
  inverse image of  Fn-14
  monotone  Fn-17
  one-line notation  Fn-2
  probability  Fn-21
  range of  Fn-2
  restricted growth and set partitions  Fn-20
  strictly decreasing  Fn-17
  strictly increasing  Fn-17
  surjective (onto)  Fn-3
  two-line notation  Fn-5
  weakly decreasing  Fn-17
  weakly increasing  Fn-17

Functional relation  Fn-4

Identity permutation  Fn-7
Image of a function  Fn-7
  Stirling numbers (set partitions) and  Fn-15
Increasing (strictly) function or list  Fn-17
Increasing (weakly) function or list  Fn-17
Index

Independent events Fn-29
Independent random variables Fn-29
Induction Fn-8
Inequality
   Tchebycheff Fn-27
Injection Fn-3
Intersection of sets Fn-1
Inverse image of a function Fn-14
Involution Fn-10

Joint distribution function Fn-28

List
   strictly decreasing Fn-17
   strictly increasing Fn-17
   weakly decreasing Fn-17
   weakly increasing Fn-17
   without repetition are injections Fn-3

Marginal distribution Fn-28
Matrix
   permutation Fn-11
Monotone function Fn-17
Multiset
   and monotone function Fn-17

Nondecreasing function or list Fn-17
Nonincreasing function or list Fn-17
Normal distribution Fn-36
Numbers
   Stirling (set partitions) Fn-15

One-line notation Fn-2
One-to-one function (injection) Fn-3

Onto function (surjection) Fn-3

Partition
   set Fn-14
   set and restricted growth function Fn-20
Permutation Fn-3, Fn-7
   cycle Fn-9
   cycle form Fn-9
   cycle length Fn-9
derangement Fn-12
identity Fn-7
involution Fn-10
is a bijection Fn-3
matrix Fn-11
powers of Fn-7
random generation Fn-33
Poisson distribution Fn-35
Probability distribution function see Distribution
Probability function Fn-21
   see also Distribution
Probability space Fn-21
   see also Distribution

Random generation of permutations Fn-33
Random variable Fn-22
   binomial Fn-34
correlation of two Fn-25
covariance of two Fn-25
independent pair Fn-29
standard deviation of Fn-25
variance of Fn-25
Range of a function Fn-2
Relation Fn-4
Restricted growth function and set partitions Fn-20

Sample space Fn-21
Index

Set
  and monotone function  Fn-17
  complement of  Fn-1
  intersection of two  Fn-1
  partition, see Set partition
  symmetric difference of
two  Fn-1
  union of two  Fn-1
Set partition  Fn-14
  restricted growth function  Fn-20
Standard deviation  Fn-25
Stirling numbers (set partitions)
  image of a function  Fn-15
Strictly decreasing function or
  list  Fn-17
Strictly increasing (or decreasing)
  function or list  Fn-17
Strictly increasing function or
  list  Fn-17
Surjection  Fn-3
Symmetric difference of sets  Fn-1

Tchebycheff’s inequality  Fn-27
Theorem
  Central Limit  Fn-38
  correlation bounds  Fn-26
  covariance when independent  Fn-32
  expectation is linear  Fn-24
  expectation of a product  Fn-32
  monotone functions and
    (multi)sets  Fn-18
  permutations of set to fixed
    power  Fn-10
  Tchebycheff’s inequality  Fn-27
  variance of sum  Fn-32
Two-line notation  Fn-5

Union of sets  Fn-1

Variance  Fn-25