

Multiple Choice Questions for Review

- In each case some information is given about a function. In which case is the information *not* sufficient to define a function?
 - $f \in \underline{4}^{\underline{3}}$, $2 \rightarrow 3$, $1 \rightarrow 4$, $3 \rightarrow 2$.
 - $f \in \{>, <, +, ?\}^{\underline{3}}$, $f = (?, <, +)$.
 - $f \in \underline{3}^{\{>, <, +, ?\}}$, $f = (3, 1, 2, 3)$.
 - $f \in \underline{3}^{\{>, <, +, ?\}}$, $f = (3, 1, 2, 3)$. Domain ordered as follows: $>$, $<$, $+$, $?$.
 - $f \in \{>, <, +, ?\}^{\underline{3}}$, $f = (?, <, +)$. Domain ordered as follows: $3, 2, 1$.
- The following function is in two line form: $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 9 & 3 & 7 & 2 & 6 & 4 & 5 & 1 & 8 \end{pmatrix}$. Which of the following is a correct cycle form for this function?
 - $(1, 8, 9)(2, 3, 7, 5, 6, 4)$
 - $(1, 9, 8)(2, 3, 5, 7, 6, 4)$
 - $(1, 9, 8)(2, 3, 7, 5, 4, 6)$
 - $(1, 9, 8)(2, 3, 7, 5, 6, 4)$.
 - $(1, 9, 8)(3, 2, 7, 5, 6, 4)$
- In each case some information about a function is given to you. Based on this information, which function is an injection?
 - $f \in \underline{6}^{\underline{5}}$, $\text{Coimage}(f) = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}\}$
 - $f \in \underline{6}^{\underline{6}}$, $\text{Coimage}(f) = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5, 6\}\}$
 - $f \in \underline{5}^{\underline{5}}$, $f^{-1}(2) = \{1, 3, 5\}$, $f^{-1}(4) = \{2, 4\}$
 - $f \in \underline{4}^{\underline{5}}$, $|\text{Image}(f)| = 4$
 - $f \in \underline{5}^{\underline{5}}$, $\text{Coimage}(f) = \{\{1, 3, 5\}, \{2, 4\}\}$
- The following function is in two line form: $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 5 & 9 & 2 & 4 & 1 & 3 & 6 & 7 \end{pmatrix}$. Which of the following is a correct cycle form for $h = f^3 \circ f^{-1}$?
 - $(1, 6, 8)(2, 3, 7)(5, 6, 4)$
 - $(1, 6, 8)(2, 4, 5)(3, 7, 9)$
 - $(1, 8, 6)(2, 3, 7)(5, 9, 4)$
 - $(1, 9, 8)(2, 3, 5)(7, 6, 4)$
 - $(8, 5, 9, 2, 4, 1, 3, 6, 7)$
- The following permutation is in two line form: $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 6 & 4 & 7 & 2 & 9 & 1 & 3 & 5 \end{pmatrix}$. The permutation $g = (1, 2, 3)$ is in cycle form. Let $h = f \circ g$ be the composition of f and g . Which of the following is a correct cycle form for h ?
 - $(1, 2, 3)(4, 5, 6, 7, 8, 9)$
 - $(1, 2, 3)(4, 5, 6, 7, 8, 9)$
 - $(1, 2, 3)(4, 5, 6, 7, 8, 9)$
 - $(1, 2, 3)(4, 5, 6, 7, 8, 9)$
 - $(1, 2, 3)(4, 5, 6, 7, 8, 9)$

Functions

- (a) (1, 6, 9, 5, 2, 4, 7)(3, 8)
 (b) (1, 8, 3, 4, 7, 2, 6)(5, 9)
 (c) (1, 8, 3, 7, 4, 2, 6)(9, 5)
 (d) (1, 8, 4, 3, 7, 2, 6)(9, 5)
 (e) (8, 6, 4, 7, 9, 1, 2)(3, 5)
6. We want to find the smallest integer $n > 0$ such that, for every permutation f on $\underline{4}$, the function f^n is the identity function on $\underline{4}$. What is the value of n ?
- (a) 4 (b) 6 (c) 12 (d) 24 (e) It is impossible.
7. In the lexicographic list of all strictly decreasing functions in $\underline{9}^{\underline{5}}$, find the successor of 98432.
- (a) 98431 (b) 98435 (c) 98521 (d) 98532 (e) 98543
8. The 16 consecutive points $0, 1, \dots, 14, 15$ have 0 and 15 converted to exterior box boundaries. The interior box boundaries correspond to points 1, 5, 7, 9. This configuration corresponds to
- (a) 9 balls into 5 boxes
 (b) 9 balls into 6 boxes
 (c) 10 balls into 5 boxes
 (d) 10 balls into 6 boxes
 (e) 11 balls into 4 boxes
9. The 16 consecutive points $0, 1, \dots, 14, 15$ have 0 and 15 converted to exterior box boundaries. The interior box boundaries correspond to the strictly increasing functions $1 \leq x_1 < x_2 < x_3 < x_4 \leq 14$ in lex order. How many configurations of balls into boxes come before the configuration $\bullet ||| \bullet \bullet$? (Exterior box boundaries are not shown.)
- (a) $\binom{13}{3}$ (b) $\binom{13}{4}$ (c) $\binom{14}{3}$ (d) $\binom{14}{4}$ (e) $\binom{15}{3}$
10. Suppose $f \in \underline{7}^{\underline{6}}$. How many such functions have $|\text{Image}(f)| = 4$?
- (a) $S(7, 4)$ (b) $S(7, 4)(6)_4$ (c) $S(6, 4)(7)_4$ (d) $S(4, 7)(6)_4$ (e) $S(7, 4) 6!$
11. Let X be a random variable with distribution $b(k; n, p)$, $q = 1 - p$. Let $Y = (X + 1)^2$. Then $E(Y) = ?$
- (a) $npq + (np + 1)^2$
 (b) $2npq + (np + 1)^2$
 (c) $npq + 2(np + 1)^2$
 (d) $(npq)^2 + (np + 1)^2$
 (e) $2npq(np + 1)^2$
12. Let X and Y be independent random variables with distribution $b(k; n, a)$ and $b(k; n, b)$ respectively. Let $Z = X + 2Y$. Then, for all $\epsilon > 0$, Tchebycheff's inequality guarantees that $P(|Z - na - 2nb| \geq \epsilon)$ is always less than or equal to what?

- (a) $(na(1 - a) + nb(1 - b))/\epsilon^2$
 (b) $(na(1 - a) + 2nb(1 - b))/\epsilon^2$
 (c) $(na(1 - a) + 4nb(1 - b))/\epsilon^2$
 (d) $(na(1 - a) + 2nb(1 - b))/\epsilon^3$
 (e) $(na(1 - a) + 4nb(1 - b))/\epsilon^3$
- 13.** An 800 page book has 400 misprints. If the misprints are distributed uniformly throughout the book, and the Poisson approximation to the binomial distribution is used to calculate the probability of exactly 2 misprints on page 16, which of the following represents the correct use of the Poisson approximation?
- (a) $e^{0.5}/8$ (b) $e^{-0.5}/8$ (c) $e^{0.5}/16$ (d) $e^{-0.5}/16$ (e) $e^{-0.5}/32$
- 14.** For 40 weeks, once per hour during the 40 hour work week, an employee of Best Cars draws a ball from an urn that contains 1 black and 9 white balls. If black is drawn, a \$10 bill is tacked to a bulletin board. At the end of the 40 weeks, the money is given to charity. What is the expected amount of money given?
- (a) 1000 (b) 1200 (c) 1400 (d) 1600 (e) 1800
- 15.** For 40 weeks, once per hour during the 40 hour work week, an employee of Best Cars draws a ball from an urn that contains 1 black and 9 white balls. If black is drawn, \$10 is tacked to a bulletin board. At the end of the 40 weeks, the money is given to charity. Using the normal approximation, what interval under the standard normal curve should be used to get the area which equals the probability that \$1800 or more is given?
- (a) from 1.67 to ∞
 (b) from 0 to 1.67
 (c) from 0.6 to ∞
 (d) from 0 to 0.6
 (e) from 0.6 to 1.67
- 16.** A fair coin is tossed three times. Let X be the random variable which is one if the first throw is T (for tails) and the third throw is H (for heads), zero otherwise. Let Y denote the random variable that is one if the second and third throws are both H , zero otherwise. The covariance, $\text{Cov}(X, Y)$ is
- (a) $1/8$ (b) $-1/8$ (c) $1/16$ (d) $-1/16$ (e) $1/32$
- 17.** A fair coin is tossed three times. Let X be the random variable which is one if the first throw is T (for tails) and the third throw is H (for heads), zero otherwise. Let Y denote the random variable that is one if the second and third throws are both H , zero otherwise. The correlation, $\rho(X, Y)$ is
- (a) 0 (b) $1/3$ (c) $-1/3$ (d) $1/8$ (e) $-1/8$
- 18.** A fair coin is tossed three times and a T (for tails) or H (for heads) is recorded, giving us a 3-long list. Let X be the random variable which is zero if no T has another T adjacent to it, and is one otherwise. Let Y denote the random variable that counts

Functions

the number of T's in the three tosses. Let $h_{X,Y}$ denote the joint distribution of X and Y . $h_{X,Y}(1, 2)$ equals

- (a) $5/8$ (b) $4/8$ (c) $3/8$ (d) $2/8$ (e) $1/8$

19. Which of the following is equal to $\text{Cov}(X + Y, X - Y)$, where X and Y are random variables on a sample space S ?

- (a) $\text{Var}(X) - \text{Var}(Y)$
(b) $\text{Var}(X^2) - \text{Var}(Y^2)$
(c) $\text{Var}(X^2) + 2\text{Cov}(X, Y) + \text{Var}(Y^2)$
(d) $\text{Var}(X^2) - 2\text{Cov}(X, Y) + \text{Var}(Y^2)$
(e) $(\text{Var}(X))^2 - (\text{Var}(Y))^2$

20. Which of the following is equal to $\text{Var}(2X - 3Y)$, where X and Y are random variables on S ?

- (a) $4\text{Var}(X) + 12\text{Cov}(X, Y) + 9\text{Var}(Y)$
(b) $2\text{Var}(X) - 3\text{Var}(Y)$
(c) $2\text{Var}(X) + 6\text{Cov}(X, Y) + 3\text{Var}(Y)$
(d) $4\text{Var}(X) - 12\text{Cov}(X, Y) + 9\text{Var}(Y)$
(e) $2\text{Var}(X) - 6\text{Cov}(X, Y) + 3\text{Var}(Y)$

21. The strictly decreasing functions in $\underline{100}^3$ are listed in lex order. How many are there before the function $(9,5,4)$?

- (a) 18 (b) 23 (c) 65 (d) 98 (e) 180

22. All but one of the following have the same answer. Which one is different?

- (a) The number of multisets of size 20 whose elements lie in $\underline{5}$.
(b) The number of strictly increasing functions from $\underline{20}$ to $\underline{24}$.
(c) The number of subsets of size 20 whose elements lie in $\underline{24}$.
(d) The number of weakly decreasing 4-lists made from $\underline{21}$.
(e) The number of strictly decreasing functions from $\underline{5}$ to $\underline{24}$.

23. Let X be a random variable with Poisson distribution $p(k; \lambda)$ Let $Y = (X + 2)(X + 1)$. What is the value of $E(Y)$?

- (a) $\lambda^2 + 3\lambda + 1$
(b) $\lambda^2 + 3\lambda + 2$
(c) $\lambda^2 + 4\lambda + 2$
(d) $3\lambda^2 + 3\lambda + 2$
(e) $4\lambda^2 + 4\lambda + 2$

Answers: **1** (c), **2** (d), **3** (a), **4** (b), **5** (a), **6** (c), **7** (c), **8** (c), **9** (a), **10** (c), **11** (a), **12** (c), **13** (b), **14** (d), **15** (a), **16** (c), **17** (b), **18** (d), **19** (a), **20** (d), **21** (c), **22** (e), **23** (c).

Notation Index

- \exists (there exists) Fn-4
- \forall (for all) Fn-4
- \ni (such that) Fn-4
- $\text{Cov}(X, Y)$ (covariance) Fn-25
- μ_X (expectation
or mean) Fn-24
- $E(X)$ (expectation) Fn-24
- $f \circ g$ (composition) Fn-7
- \underline{n} (first n integers) Fn-1
- $\mathcal{P}_k(A)$ (k -subsets of A) Fn-1
- $\mathcal{S}(A)$ (permutations of A) Fn-7
- $\text{PER}(A)$ (permutations of A) Fn-7
- Probability notation
 - μ_X (expectation, or
mean) Fn-24
 - $\rho(X, Y)$ (correlation) Fn-25
 - σ_X (standard deviation) Fn-25
 - $E(X)$ (expectation) Fn-24
 - $\text{Cov}(X, Y)$ (covariance) Fn-25
 - $\text{Var}(X)$ (variance) Fn-25
- \mathbb{Q} (rational numbers) Fn-1
- \mathbb{R} (real numbers) Fn-1
- $\rho(X, Y)$ (correlation) Fn-25
- Set notation
 - $\sim A$ (complement) Fn-1
 - A' (complement) Fn-1
 - $A - B$ (difference) Fn-1
 - $A \cap B$ (intersection) Fn-1
 - $A \cup B$ (union) Fn-1
 - $A \oplus B$ (symmetric
difference) Fn-1
 - $A \setminus B$ (difference) Fn-1
 - $A \times B$ (Cartesian product) Fn-1
 - A^c (complement) Fn-1
 - $\mathcal{P}_k(A)$ (k -subsets of A) Fn-1
- σ_X (standard deviation) Fn-25
- $\text{Var}(X)$ (variance) Fn-25
- \mathbb{Z} (integers) Fn-1

Subject Index

- Bijection Fn-3
- Binomial distribution Fn-34
- Blocks of a partition Fn-15

- Cartesian product Fn-1
- Central Limit Theorem Fn-38
- Chebyshev's inequality Fn-27
- Codomain (range) of a function Fn-2
- Coimage of a function Fn-14
- Complement of a set Fn-1
- Composition of functions Fn-7
- Correlation Fn-25
- Covariance Fn-25
- Cycle in a permutation Fn-9

- Decreasing (strictly) function or list Fn-17
- Decreasing (weakly) function or list Fn-17
- Density function Fn-22
- Derangement Fn-12
- Deviation
 - standard Fn-25
- Direct (Cartesian) product Fn-1
- Distribution Fn-22
 - binomial Fn-34
 - joint Fn-28
 - marginal Fn-28
 - normal Fn-36
 - Poisson Fn-35
- Distribution function
 - see* Distribution
- Domain of a function Fn-2

- Envelope game Fn-2

- Event Fn-21
 - independent pair Fn-29
- Expectation of a random variable Fn-24

- Function
 - bijection Fn-3
 - codomain (range) of Fn-2
 - coimage of Fn-14
 - composition of Fn-7
 - density Fn-22
 - distribution, *see* Distribution
 - domain of Fn-2
 - image of Fn-14
 - image of and Stirling numbers (set partitions) Fn-15
 - injective (one-to-one) Fn-3
 - inverse Fn-3
 - inverse image of Fn-14
 - monotone Fn-17
 - one-line notation Fn-2
 - probability Fn-21
 - range of Fn-2
 - restricted growth and set partitions Fn-20
 - strictly decreasing Fn-17
 - strictly increasing Fn-17
 - surjective (onto) Fn-3
 - two-line notation Fn-5
 - weakly decreasing Fn-17
 - weakly increasing Fn-17
- Functional relation Fn-4

- Identity permutation Fn-7
- Image of a function Fn-14
 - Stirling numbers (set partitions) and Fn-15
- Increasing (strictly) function or list Fn-17
- Increasing (weakly) function or list Fn-17

Index

- Independent events Fn-29
- Independent random
 - variables Fn-29
- Induction Fn-8
- Inequality
 - Tchebycheff Fn-27
- Injection Fn-3
- Intersection of sets Fn-1
- Inverse image of a function Fn-14
- Involution Fn-10

- Joint distribution function Fn-28

- List
 - strictly decreasing Fn-17
 - strictly increasing Fn-17
 - weakly decreasing Fn-17
 - weakly increasing Fn-17
 - without repetition are
 - injections Fn-3

- Marginal distribution Fn-28
- Matrix
 - permutation Fn-11
- Monotone function Fn-17
- Multiset
 - and monotone function Fn-17

- Nondecreasing function or
 - list Fn-17
- Nonincreasing function or list Fn-17
- Normal distribution Fn-36
- Numbers
 - Stirling (set partitions) Fn-15

- One-line notation Fn-2
- One-to-one function (injection) Fn-3

- Onto function (surjection) Fn-3

- Partition
 - set Fn-14
 - set and restricted growth
 - function Fn-20
- Permutation Fn-3, Fn-7
 - cycle Fn-9
 - cycle form Fn-9
 - cycle length Fn-9
 - derangement Fn-12
 - identity Fn-7
 - involution Fn-10
 - is a bijection Fn-3
 - matrix Fn-11
 - powers of Fn-7
 - random generation Fn-33
- Poisson distribution Fn-35
- Probability distribution function
 - see* Distribution
- Probability function Fn-21
 - see also* Distribution
- Probability space Fn-21
 - see also* Distribution

- Random generation of
 - permutations Fn-33
- Random variable Fn-22
 - binomial Fn-34
 - correlation of two Fn-25
 - covariance of two Fn-25
 - independent pair Fn-29
 - standard deviation of Fn-25
 - variance of Fn-25
- Range of a function Fn-2
- Relation Fn-4
- Restricted growth function and set
 - partitions Fn-20

- Sample space Fn-21

- Set
 - and monotone function Fn-17
 - complement of Fn-1
 - intersection of two Fn-1
 - partition, *see* Set partition
 - symmetric difference of
 - two Fn-1
 - union of two Fn-1
- Set partition Fn-14
 - restricted growth function Fn-20
- Standard deviation Fn-25
- Stirling numbers (set partitions)
 - image of a function Fn-15
- Strictly decreasing function or
 - list Fn-17
- Strictly increasing (or decreasing)
 - function or list Fn-17
- Strictly increasing function or
 - list Fn-17
- Surjection Fn-3
- Symmetric difference of sets Fn-1

- Tchebycheff's inequality Fn-27
- Theorem
 - Central Limit Fn-38
 - correlation bounds Fn-26
 - covariance when independent Fn-32
 - expectation is linear Fn-24
 - expectation of a product Fn-32
 - monotone functions and
 - (multi)sets Fn-18
 - permutations of set to fixed
 - power Fn-10
 - Tchebycheff's inequality Fn-27
 - variance of sum Fn-32
- Two-line notation Fn-5

- Union of sets Fn-1

- Variance Fn-25

- Weakly decreasing function or
 - list Fn-17
- Weakly increasing function or
 - list Fn-17