Proof of perceptron algorithm:


Beware! The proof uses very bad notation (using various characters over and over again, even in the same equation!)

Bound of perceptron algorithm:

http://www.cse.iitd.ernet.in/~naveen/courses/CS758/3 The Perceptron Algorithm.doc

Conditional probability definition:

Conditional probability is the probability of some event $A$, given the occurrence of some other event $B$. Conditional probability is written $P(A|B)$, and is read "the probability of $A$, given $B$".

Conditional probability fallacy:

The conditional probability fallacy is the assumption that $P(A|B)$ is approximately equal to $P(B|A)$. The mathematician John Allen Paulos discusses this in his book Innumeracy (p. 63 et seq.), where he points out that it is a mistake often made even by doctors, lawyers, and other highly educated non-statisticians. It can be overcome by describing the data in actual numbers rather than probabilities.

The relation between $P(A|B)$ and $P(B|A)$ is given by Bayes Theorem: $P(B|A)=P(A|B)*P(B)/P(A)$. In other words, one can only assume that $P(A|B)$ is approximately equal to $P(B|A)$ if the prior probabilities $P(A)$ and $P(B)$ are also approximately equal.

Conditional probability fallacy example:

In the following constructed but realistic situation, the difference between $P(A|B)$ and $P(B|A)$ may be surprising, but is at the same time obvious.

In order to identify individuals having a serious disease in an early curable form, one may consider screening a large group of people. While the benefits are obvious, an
argument against such screenings is the disturbance caused by false positive screening results: If a person not having the disease is incorrectly found to have it by the initial test, they will most likely be quite distressed until a more careful test shows that they do not have the disease. Even after being told they are well, their lives may be affected negatively.

The magnitude of this problem is best understood in terms of conditional probabilities.

Suppose 1% of the group suffer from the disease, and the rest are well. Choosing an individual at random,

\[ P(\text{disease}) = 1\% = 0.01 \text{ and } P(\text{well}) = 99\% = 0.99. \]

Suppose that when the screening test is applied to a person not having the disease, there is a 1% chance of getting a false positive result, i.e.

\[ P(\text{positive} | \text{well}) = 1\%, \text{ and } (\text{negative} | \text{well}) = 99\%. \]

Finally, suppose that when the test is applied to a person having the disease, there is a 1% chance of a false negative result, i.e.

\[ P(\text{negative} | \text{disease}) = 1\% \text{ and } P(\text{positive} | \text{disease}) = 99\%. \]

Now, one may calculate the following:

The fraction of individuals in the whole group who are well and test negative:

\[ P(\text{well} \cap \text{negative}) = P(\text{well}) \times P(\text{negative} | \text{well}) = 99\% \times 99\% = 98.01\%. \]

The fraction of individuals in the whole group who are ill and test positive:

\[ P(\text{disease} \cap \text{positive}) = P(\text{disease}) \times P(\text{positive} | \text{disease}) = 1\% \times 99\% = 0.99\%. \]

The fraction of individuals in the whole group who have false positive results:

\[ P(\text{well} \cap \text{positive}) = P(\text{well}) \times P(\text{positive} | \text{well}) = 99\% \times 1\% = 0.99\%. \]
The fraction of individuals in the whole group who have false negative results:

\[ P(\text{disease} \cap \text{negative}) = P(\text{disease}) \times P(\text{negative} | \text{disease}) = 1\% \times 1\% = 0.01\%. \]

Furthermore, the fraction of individuals in the whole group who test positive:

\[
P(\text{positive}) = P(\text{well} \cap \text{positive}) + P(\text{disease} \cap \text{positive})
= 0.99\% + 0.99\% = 1.98\%.
\]

Finally, the probability that an individual actually has the disease, given that the test result is positive:

\[
P(\text{disease} | \text{positive}) = \frac{P(\text{disease} \cap \text{positive})}{P(\text{positive})} = \frac{0.99\%}{1.98\%} = 50\%.
\]

In this example, it should be easy to relate to the difference between the conditional probabilities \( P(\text{positive} | \text{disease}) \) (which is 99%) and \( P(\text{disease} | \text{positive}) \) (which is 50%): the first is the probability that an individual who has the disease tests positive; the second is the probability that an individual who tests positive actually has the disease. With the numbers chosen here, the last result is likely to be deemed unacceptable: half the people testing positive are actually false positives.