Section 5 Notes, February 6, 2004, by Kristin Branson

1 FOL Terminology Review

The basic syntactic elements of first-order logic (FOL) are the names of objects, relationships, and functions. A *constant symbol* is a name for an object. A *predicate symbol* is a name for the relationship between objects ($=$ is a special predicate-symbol). A *function symbol* is a name for a function that maps objects to objects. A *variable* is a special identifier that stands for an arbitrary object. A *term* is any logical expression that refers to an object (a constant symbol, a variable, of a function of a term). We can express properties of collections of objects using the *universal quantifier* ($\forall$) and the *existential quantifier* ($\exists$). Our knowledge base consists of sentences, which correspond to statements of facts.

There are many possible universes of objects, and for each universe there are many possible relationships and functions. Each of these alternative worlds is called an *interpretation*. A *model* is an interpretation in which all axioms are true. The *intended interpretation* is the interpretation in mind when the symbols were determined.

2 Example 1: Passing the Midterm

Suppose we wish to predict whether a given student will pass the midterm on 02/17. What is required of a student in order for him to pass? There are a couple of alternatives:

- The student is prepared for the midterm.
- The midterm is easy.

What is required of a student, in order to be prepared for an midterm? Again, there are a few alternatives:

- The student already knows everything relevant.
- The student has studied all quarter.
- The student was tutored on the relevant material.

In order to be studied, a student must have read the textbook, attended lectures, and done the homeworks. We now must translate these commonsense facts into syntactic sentences. What symbols should we use? Let's choose the following predicates: $pass\_midterm(\cdot)$, $knows\_it\_all(\cdot)$, $studied(\cdot)$, $tutored\_by(\cdot,\cdot)$, $read\_textbook(\cdot)$, $attended\_lecture(\cdot)$, $did\_homework(\cdot)$. We also have the predicate $easy\_midterm$ which takes no inputs. What is the intended interpretation of these symbols?

- A student is in the set $pass\_midterm(\cdot)$ if we predict that the student will pass the midterm.
- A student is in the set $knows\_it\_all(\cdot)$ if the student already knows all the relevant material.
- A student is in the set $studied(\cdot)$ if the student has studied all quarter.
- A pair of people are in the set $tutored\_by(\cdot,\cdot)$ if the first person was tutored by the second person.
- A student is in the set $read\_textbook(\cdot)$ if he read the required textbook for the course.
- A student is in the set $attended\_lecture(\cdot)$ if he attended lecture all quarter.
- A student is in the set $did\_homework(\cdot)$ if he did all the homework assignments for the class.
- The predicate $easy\_midterm$ is true if the midterm is easy, and false otherwise.

Here is a set of axioms encoding the above facts:

- An object will pass the midterm if it is prepared or the midterm is easy:

$$\forall x \quad (prepared\_for\_midterm(x) \vee easy\_midterm) \rightarrow pass\_midterm(x).$$

- An object is prepared for the midterm if it already knows everything relevant, or if the object studied all quarter, or if the object was tutored:

$$\forall x \quad (knows\_it\_all(x) \vee studied(x) \vee (\exists y \quad tutored\_by(x,y))) \rightarrow prepared\_for\_midterm(x).$$

- An object has studied all quarter if it has read the textbook, attended lecture, and done the homework:

$$\forall x \quad (read\_textbook(x) \wedge attended\_lecture(x) \wedge did\_homework(x)) \rightarrow studied(x).$$

To check to see if our intended interpretation is a model of this knowledge base, let us try some example problem instances, and see if we get the expected results.

- Suppose our problem instance consists of the sentences $\neg easy\_exam$ and $knows\_it\_all(ProfessorElkan)$. Is $pass\_midterm(ProfessorElkan)$ entailed by our knowledge base? Yes, since, if Professor Elkan knows all the relevant material, then he is prepared for the midterm, and will pass.

- Suppose our problem instance consists of the sentences $\neg easy\_exam$ and $tutored\_by(Alice, PresidentBush)$. Is $pass\_midterm(Alice)$ entailed by our knowledge base? Yes, because $prepared\_for\_midterm(Alice)$ is $true$. Does this reflect our intended interpretation? No, being tutored by President Bush would not help Alice pass the midterm. We must change an axiom to state that a tutor must have studied the relevant material. We could change the axiom to be

$$\forall x \quad (knows\_it\_all(x) \vee studied(x) \vee (\exists y \quad studied(y) \wedge tutored\_by(x, y))) \rightarrow prepared\_for\_midterm(x).$$

Now, if we add the sentence $\neg studied(PresidentBush)$, then $pass\_midterm(Alice)$ is not entailed by our knowledge base.

- Suppose the following problem instance:
  - $\neg easy\_exam.$
  - $tutored\_by(Alice, ProfessorElkan).$
  - $knows\_it\_all(ProfessorElkan).$
  - $\neg did\_homework(ProfessorElkan)$

  Given the new set of axioms, is $pass\_midterm(Alice)$ entailed by the knowledge base? No, it is not, since $studied(ProfessorElkan) = false$. This does not reflect the intended interpretation. We should make the changed axiom less restrictive:

$$\forall x \quad (knows\_it\_all(x) \vee studied(x) \vee (\exists y \quad prepared\_for\_midterm(y) \wedge tutored\_by(x, y))) \rightarrow prepared\_for\_midterm(x).$$

- Suppose our problem instance consists of the sentence $easy\_midterm$, is $pass\_midterm(MyPetSeaMonkey)$ entailed by the knowledge base? Yes, it is. This also does not represent the intended model. We must make the first axiom more restrictive:

$$\forall x \quad (prepared\_for\_midterm(x) \vee (easy\_midterm \wedge UCSD\_student(x))) \rightarrow pass\_midterm(x).$$

  Now, if we include the sentence $\neg UCSD\_student(MyPetSeaMonkey)$, then the sentence is not entailed by the knowledge base.

3 Example 2: Animal Taxonomy

In this example, we want to create a knowledge base about animals. We would like to make queries to this knowledge base. For instance, we might want to see whether a tiger is a mammal. Our knowledge consists of the following statements:

- Tigers, ostriches, salmon, and pelicans are animal species.
- Birds, fish, and mammals are classes of animals.
- Animals that fly are birds.
- Fish swim.
- Animals that have hair are mammals.
- Animals that have striped fur are tigers.
- Ostriches have feathers.
- Salmon swim.
- Pelicans fly.

We must first decide on our ontology. We will have a function $species(\cdot)$ that returns what species the input term is. We will have a function $class(\cdot)$ that returns what animal class the input term is. We will have a predicate for each of the properties described: $is\_species(\cdot)$, $is\_class(\cdot)$, $fly(\cdot)$, $swim(\cdot)$, $have\_hair(\cdot)$, $striped(\cdot)$, $have\_fur(\cdot)$, and $have\_feathers(\cdot)$. We will have a constant for each of the types of species: tiger, ostrich, salmon, and pelican. We will have a constant for each type of class: bird, fish, and mammal.

We now must write axioms that encode the above knowledge:

- Tigers, ostriches, salmon, and pelicans are animal species:

$$is\_species(tiger) \wedge is\_species(ostrich) \wedge is\_species(salmon) \wedge is\_species(pelican).$$

- Birds, fish, and mammals are classes of animals:

$$is\_class(bird) \wedge is\_class(fish) \wedge is\_class(animal).$$

- Animals that fly are birds:

$$\forall x \quad fly(x) \rightarrow (class(x) = bird).$$

- Fish swim:

$$\forall x \quad (class(x) = fish) \rightarrow swim(x).$$

- Animals that have hair are mammals:

$$\forall x \quad have\_hair(x) \to (class(x) = mammal).$$

- Animals that have striped fur are tigers:

$$\forall x \quad (have\_fur(x) \wedge striped(x)) \to (species(x) = tiger).$$

- Ostriches have feathers:

$$\forall x \quad (species(x) = ostrich) \to have\_feathers(x).$$

- Salmon swim:

$$\forall x \quad (species(x) = salmon) \to swim(x).$$

- Pelicans fly:

$$\forall x \quad (species(x) = pelican) \to fly(x).$$

Let us now test our knowledge base on some example problem instances, to see if our intended interpretation is a model of the knowledge base.

- Consider the sentence $\forall x \quad (species(x) = salmon) \to (class(x) = fish)$. Does our knowledge base entail this sentence? No, because of the direction of the implications in the axioms. Just because an object swims, it does not imply that it is a fish (for example, consider a dolphin or a duck). We must add some more details to our knowledge base. For instance, we could add the fact that all fish have gills and all animals with gills are fish:

  $$\forall x \quad have\_gills(x) \leftrightarrow (class(x) = fish).$$

  Then, we could add the fact that salmon have gills:

  $$\forall x \quad (species(x) = salmon) \to have\_gills(x).$$

- Suppose we evaluate the sentence $\forall x \quad (species(x) = ostrich) \to (class(x) = bird)$. This sentence is not entailed by the knowledge base, since all we know is that animals that fly are birds. We could fix our knowledge base by changing an axiom to express that animals with feathers are birds:

  $$\forall x \quad have\_feathers(x) \to (class(x) = bird).$$

  Now, since we know that ostriches have feathers, and objects with feathers are birds, our sentence is entailed by the knowledge base.

- Now suppose we evaluate the sentence $\forall x \quad (species(x) = pelican) \to (class(x) = bird)$. This sentence is now not entailed by the knowledge base, since now the only characteristic about birds is that they include animals with feathers. We could fix our knowledge base by adding an axiom, interpreted as expressing that if an animal flies then it must have feathers:

  $$\forall x \quad fly(x) \to have\_feathers(x).$$

  Now, since we know that pelicans fly, and objects that fly have feathers, and objects with feathers are birds, our sentence is entailed by the knowledge base.

- Suppose we evaluate the sentence $\forall x \quad (class(x) = bird) \to have\_feathers(x)$. This sentence is not entailed by our knowledge base, because of the direction of the implication. Thus, our intended interpretation is not a model of the knowledge base. We could fix this by changing the implication to an equivalence:

  $$\forall x \quad have\_feathers(x) \leftrightarrow (class(x) = bird).$$

  Now, our sentence is entailed by the knowledge base.

- Suppose we evaluate the sentence $\forall x \quad (species(x) = tiger) \to (class(x) = mammal)$. This sentence is not entailed by our knowledge base. This is because our knowledge base does not contain the fact that fur is a type of hair. This does not reflect the intended interpretation. We could fix this by adding the axiom:

  $$\forall x \quad have\_fur(x) \to have\_hair(x).$$

  Now, since we know that all tigers have fur and all objects with fur have hair, and all objects with hair are mammals, our sentence is entailed by the knowledge base.