An adaptive nearest neighbor rule for classification

Akshay Balsubramani, Stanford
Sanjoy Dasgupta, UCSD
Yoav Freund, UCSD
Shay Moran, Google AI Princeton
Main Idea: Modify $k$-NN Algorithm by Choosing $k$ Adaptively for Each Query

- **Classical $k$-NN**: classify $x$ by the majority vote of its $k$ nearest in the training set.

$x$ is the green point in the middle. The label assigned to $x$ is determined by its $k$ nearest neighbors (inside the big circle, in this example $k=13+12=25$)
Main Idea: Modify $k$-NN Algorithm by Choosing $k$ Adaptively for Each Query

- **Adaptive $k$-NN:**
  - Iterate over the neighbors of $x$ from nearest to furthest and query their labels.
  - If one of the label-classes obtains a significant majority then exit the loop and use this label to classify $x$.

Points $x$ that are close to the boundary require querying a large number of neighbors.

Points $x$ that are far from the boundary observe a significant advantage after querying a small number of neighbors.
Main Results

Theoretical Results

1. Adaptive k-NN rule is consistent (i.e. achieves Bayes optimality in the limit).

2. Instance-dependent generalization bounds
   - Number of examples required to classify $x$ correctly depends on its “local-margin” (a formal notion introduced in the paper).
   - Points far from the boundary are correctly classified fast.

Practical Results

1. Adaptive $k$-NN rule is competitive with Classical $k$-NN with the best choice of $k$
   - Thus, this method circumvents the need to tune the meta-parameter $k$. 