Homework Nine, for Wed 11/23

CSE 101

1. Alice wants to throw a party and is deciding whom to call. She has n people to choose from, and she has made up a list of which pairs of these people know each other. She wants to pick as many people as possible, subject to a constraint: at the party, each person should have at least five other people whom they know.

   (a) Give a high-level description of an efficient algorithm that takes as input the list of n people and the list of pairs who know each other and outputs the best choice of party invitees. Argue carefully that this scheme is correct.

   (b) Give an efficient implementation of the scheme from part (a), and analyze its running time in terms of n.

   (c) Suppose that Alice adds another constraint: each person should have at least five other people whom they know *and* five other people whom they don't know. Give an algorithm for this case as well.

2. You are driving down a very long highway, with gas stations at mile-posts $m_1, m_2, \ldots, m_n$, where $m_1 = 0$ is your starting point and m_n is your final destination. You want to make as few gas stops as possible, but your car can only hold enough gas to cover M miles. Give an algorithm to find the minimum number of stops you need to make. Justify the correctness of the algorithm, and analyze its running time.

3. *Another scheduling problem.* 5.32.

4. We are given a collection of intervals on the line: $I_1 = [\ell_1, u_1], \ldots, I_n = [\ell_n, u_n]$. We'd like to select a small set of points on the line, such that each interval contains at least one of the points. Give an efficient algorithm for finding the smallest possible set of such points, and justify its correctness.

5. *Understanding trees.* Problem 5.4 from the book.

6. *Minimum spanning trees example.* 5.1.