Math 96:
Practice Problems

November 2nd, 2018

1988 B1 A composite positive integer is a product \( ab \) with \( a \) and \( b \) not necessarily distinct integers in \( \{2, 3, 4, \ldots\} \). Show that every composite integer is representable as \( xy + yz + zx + 1 \) with \( x, y \) and \( z \) positive integers.

2008 B1 What is the maximum number of rational points that can lie on a circle in \( \mathbb{R}^2 \) whose center is not a rational point?

2008 A1 Let \( f : \mathbb{R}^2 \to \mathbb{R} \) be a function so that \( f(x, y) + f(y, z) + f(z, x) = 0 \) for all real numbers \( x, y \) and \( z \). Prove that there exists a function \( g : \mathbb{R} \to \mathbb{R} \) so that \( f(x, y) = g(x) - g(y) \) for all real numbers \( x \) and \( y \).