This homework is due in class on Friday, October 19th. Please complete one of the following problems.

1958-Winter B2 Given a set of $n + 1$ integers, none of which exceeds $2n$, show that at least one member of the set must divide another member of the set.

2013 A1 Recall that the regular icosahedron is a convex polyhedron having 12 vertices and 20 faces; the faces are congruent equilateral triangles. On each face of a regular icosahedron is drawn a nonnegative integer so that the sum of the 20 integers is 39. Show that there are two faces that share a vertex and have the same integer written on them.

2013 B3 Let $\mathcal{P}$ be a nonempty collection of subsets of $\{1, 2, \ldots, n\}$ such that

1. If $S, S' \in \mathcal{P}$, then $S \cup S' \in \mathcal{P}$ and $S \cap S' \in \mathcal{P}$, and
2. If $S \in \mathcal{P}$ and $S \neq \emptyset$, then there is a subset $T \subset S$ such that $T \in \mathcal{P}$ and $T$ contains exactly one fewer element than $S$.

Suppose that $f : \mathcal{P} \to \mathbb{R}$ is a function such that $f(\emptyset) = 0$ and

$$f(S \cup S') = f(S) + f(S') - f(S \cap S')$$

for all $S, S' \in \mathcal{P}$.

Must there exist real numbers $f_1, f_2, \ldots, f_n$ so that

$$f(S) = \sum_{i \in S} f_i$$

for all $S \in \mathcal{P}$?