Math 96:
Homework 1

Fall 2017

This homework is due in class on Friday, October 6th. Please complete one of the following problems.

2012 B1 Let $S$ be a class of functions from $[0, \infty)$ to $[0, \infty)$ that satisfies:

1. The functions $f_1(x) = \log(1 + x)$ and $f_2(x) = e^x - 1$ are in $S$;
2. If $f(x)$ and $g(x)$ are in $S$ then $f(x) + g(x)$ and $f(g(x))$ are in $S$;
3. If $f(x)$ and $g(x)$ are in $S$ and $f(x) \geq g(x)$ for all $x \geq 0$, then $f(x) - g(x)$ is in $S$.

Prove that if $f(x)$ and $g(x)$ are in $S$ then the function $f(x)g(x)$ is also in $S$.

1987 A2 The sequence of digits

12345678910111213141516171819202122\ldots

is obtained by writing the positive integers in order. If the $10^n$-th digit in this sequence occurs in the part of the sequence where the $m$-digit numbers are placed, define $f(n)$ to be $m$. For example, $f(2) = 2$ since the $100^{th}$ digit enters the sequence in the placement of the two-digit number 55. Find with proof $f(1987)$.

1967 A3 Consider polynomial forms $ax^2 - bx + c$ with integer coefficients which have two distinct zeroes in the open interval $0 < x < 1$. Exhibit with a proof the least positive integer value of $a$ for which such a polynomial exists.