Math 96:
Homework 1

Fall 2018

This homework is due in class on Friday, October 5th. Please complete one of the following problems.

1988 A1 Let $R$ be the region of points $(x, y)$ in cartesian plane satisfying both $|x| - |y| \leq 1$ and $|y| \leq 1$. Sketch the region $R$ and find its area.

2003 A1 Let $n$ be a fixed positive integer. How many ways are there of writing $n$ as a sum of positive integers $n = a_1 + a_2 + \ldots + a_k$ with $k$ an arbitrary positive integer and $a_1 \leq a_2 \leq \ldots \leq a_k \leq a_1 + 1$? For example, when $n = 4$ there are four ways: $4, 2 + 2, 1 + 1 + 2, 1 + 1 + 1 + 1$.

2013 B1 For positive integers $n$ let $c(n)$ be determined by the rules $c(1) = 1$, $c(2n) = c(n)$, and $c(2n + 1) = (-1)^n c(n)$. Find the value of

$$\sum_{n=1}^{2013} c(n)c(n + 2).$$

1993 B3 Two real numbers $x$ and $y$ are chosen at random from the interval $(0, 1)$ with respect to the uniform distribution. What is the probability that the closest integer to $x/y$ is even? Express your answer in the form $r + s\pi$ where $r$ and $s$ are rational numbers.