So you have a new problem to solve. Especially early on, it can be confusing as to what exactly to do with it. Here are some basic techniques to get started.

1 Read the Problem

No, seriously. Read the problem carefully.

- Do you know the meaning of all the words? If you do not, you might just need to skip this problem (a few Putnam problems will involve more advanced math).

- Can you restate the problem in your own words?

- Can you list all of the assumptions that you are allowed to make? Note: very often you will need to make use of every assumption in your proof.

2 Things to Try

Once you understand the problem, you then need to try to find a solution. Mostly, this involves stumbling around and trying new things until you either gain some insight or find something that works. Perhaps the single best piece of advice I can give you is this:

**Don’t give up.** As long as you are trying new things, even if they don’t work, you are making progress.

But sometimes you get stuck and don’t know what to do. If you don’t know what to try next, here are some general techniques to fall back on. Not all of these will apply to all problems, but for many problems at least one of these will give you a good start in whatever you are working on.

- Draw a picture.

- Try working small examples or special cases.

- Look for patterns.
• Guess and check (use induction).
• Give variable names to relevant quantities.
• See what you can compute or prove.
• Write down formulas. See what you can derive from them.
• Make conjectures. What seems to be true about this problem. Can you prove it?
• Work backwards. What would imply the problem statement? What else needs to be true if the problem statement is?
• Use proof by contradiction. Try to construct a counterexample- it won’t work, but why it doesn’t work is usually instructive.
• Reread the problem statement. Are there conditions that you aren’t using?
• Use symmetry.
• Consider extremal cases. What can you say about the furthest away/largest/most whatever object in the problem?

3 Examples

1997 A2 Players 1, 2, 3, . . . , n are seated around a table, and each has a single penny. Player 1 passes a penny to player 2 who then passes two pennies to player 3. Player 3 passes one penny to player 4, who then passes two pennies to player 5, and so on, players alternatively passing one penny or two to the next player who still has some pennies. A player who runs out of pennies drops out of the game and leaves the table. Find an infinite set of numbers n for which some player ends up with all n pennies.

1992 A1 Prove that \( f(n) = n - 1 \) is the only integer-valued function defined on the integers so that

1. \( f(f(n)) = n \) for all integers n;
2. \( f(f(n + 2) + 2) = n \) for all integers n;
3. \( f(0) = 1 \).

1942 A1 A square of side 2a, lying always in the first quadrant of the XY plane, moves so that two consecutive vertices are always on the X- and Y- axes, respectively. Find the locus of the midpoint of the square.