Math 184A Homework 7

Spring 2018

This homework is due on gradescope by Friday June 8th at 11:59pm. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in \LaTeX is recommended though not required.

**Question 1** (Avoidance Bounds, 20 points). From the book we know that $S_n(1432) \leq 9^n$. Find a constant $C$ so that $S_n(321456987) \leq C^n$ for all $n$.

**Question 2** (Hill Avoidance, 40 points). Let a $k$-hill in a permutation be a subsequence of $2k-1$ of the entries the first $k$ of which are in increasing order and the last $k$ of which are in decreasing order. Note that a $k$-hill is not a single pattern. For example, a 2-hill is either an instance of the pattern 132 or an instance of the pattern 231.

(a) Show that the number of permutations of $[n]$ with no 2-hill is $2^{n-1}$. [15 points]

(b) Show that the number of permutations of $[n]$ with no $k$-hill is at most $(4(k-1)^2)^n$. [Hint: try to find a decreasing sequence among elements that are the largest of a $k$-term increasing subsequence.] [25 points]

**Question 3** (Marriage Lemma, 40 points). The Marriage Lemma states that if you are given two sets $S$ and $T$ of size $n$ and a set $E$ of pairs of one element of each set, then there is a matching between $S$ and $T$ (namely a set of $n$ pairs from $E$ using each element of $S$ and each element of $T$ exactly once) unless there is some subset $S' \subset S$ so that the total number of elements of $T$ that pair with some element of $S'$ is less than $|S'|$.

Prove the Marriage Lemma using Dilworth’s Theorem.

**Question 4** (Extra credit, 1 point). Free point!