Math 184A Homework 1

Fall 2016

This homework is due on gradescope by Friday October 7th at 11:59pm. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in $\LaTeX$ is recommended though not required.

**Question 1** (Fibonacci Numbers, 30 points). *The Fibonacci Numbers is the sequence* $1, 1, 2, 3, 5, 8, 13 \ldots$ *defined as follows:*

\[ F_1 = F_2 = 1. \]
\[ F_{n+2} = F_{n+1} + F_n \text{ for all } n \geq 1. \]

(a) Prove that $F_{n+2}$ counts the number of strings of length $n$ using letters ‘A’ and ‘B’ so that there are no two consecutive A’s. So for example “ABBAB” would be a valid string of length 5, but not “AABBA”.

[15 points]

(b) Prove by induction that

\[ F_n = \frac{1}{\sqrt{5}} \left( \left( \frac{1 + \sqrt{5}}{2} \right)^n - \left( \frac{1 - \sqrt{5}}{2} \right)^n \right) \]

for all $n \geq 1$. Hint: when using induction you will need to handle the case of $n = 2$ as a second base case. [15 points]

**Question 2** (Course Overlap, 30 points). *In Binomial University, there are a total of 32 classes taught each term and each student takes exactly 5 classes each term. Show that for any group of 45 students that some two of them must share a pair of classes in common. Hint: First show that there must be some class taken by at least 8 students.*

**Question 3** (Counting Words, 40 points). *For the purposes of this problem, a word is simply a sequence of letters from \{a, b, \ldots, z\}, the vowels are a, e, i, o, u. How many words are there of each of the following types (you can give answers as unexpanded expressions, like \binom{26}{8}, but you should still justify your answers) [5 points each]:*

(a) With either exactly 4 or exactly 5 letters.

(b) With at most 5 letters, all of which are the same.

(c) With exactly 5 letters, the first two of which are vowels.

(d) With exactly 5 letters, all of which are distinct.

(e) With exactly 5 letters, all distinct, appearing in alphabetical order.

(f) With exactly 5 letters, at least one of which is a vowel.

(g) With exactly 6 letters, exactly two of which are the same.

(h) With exactly 5 letters and with all the vowels in the word coming before all of the non-vowels.

**Question 4** (Extra credit, 1 point). *Approximately how much time did you spend working on this homework?*