Instructions: Do not open until the exam starts. The exam will run for 45 minutes. The problems are roughly sorted in increasing order of difficulty. Answer all questions completely. In particular, in order to get full credit, you will need to provide a proof of your results. You are free to make use of any result in the textbook or proved in class. You may use up to 6 1-sided pages of notes, and may not use the textbook nor any electronic aids. Write your solutions in the space provided, the pages at the end of this handout, or on the scratch paper provided (be sure to label it with your name). If you have solutions written anywhere other than the provided space be sure to indicate where they are to be found.

Please be sure to sit in the seat indicated below for the exam.

Name:

ID Number:

Seat:

Discussion Section: (please circle one)

A01 8-9pm, AP&M B402A (Dun Qiu)
A02 5-6pm, AP&M B402A (Kyle Meyer)
A03 6-7pm, AP&M B402A (Kyle Meyer)
A04 7-8pm, AP&M B402A (Dun Qiu)
A05 6-7pm, CENTR 201 (Scott Fernandez)
A06 7-8pm, CENTR 201 (Scott Fernandez)

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1
Question 1 (Generating Function Computation, 30 points). What are the first 5 coefficients (the $x^0$ coefficient through that $x^4$ coefficient) of the generating function
\[
\frac{\sqrt{1 + 2x^2}}{(1 + x)^2}.
\]
Question 2 (No Hamiltonian Paths, 35 points). Show that if $G$ is a graph with at least three vertices of degree 1, that $G$ does not contain any Hamiltonian paths.
Question 3 (Derangement Equation, 35 points). Show that

\[ n! = \sum_{k=0}^{n} \binom{n}{k} D_{n-k} \]

where \( D_m \) is the number of derangements on \([m]\).