Instructions: Do not open until the exam starts. The exam will run for 45 minutes. The problems are roughly sorted in increasing order of difficulty. Answer all questions completely. In particular, in order to get full credit, you will need to provide a proof of your results. You are free to make use of any result in the textbook or proved in class. You may use up to 6 1-sided pages of notes, and may not use the textbook nor any electronic aids. Write your solutions in the space provided, the pages at the end of this handout, or on the scratch paper provided (be sure to label it with your name). If you have solutions written anywhere other than the provided space be sure to indicate where they are to be found.

Please be sure to sit in the seat indicated below for the exam.

Name:

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ID Number:

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Discussion Section:

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Seat:

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<table>
<thead>
<tr>
<th>Problem</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Score</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Question 1 (Self-Conjugate Partitions, 30 points). Consider the partition of 13 into distinct odd parts given by $9 + 3 + 1 = 13$. What is the corresponding self-conjugate partition of 13?
Question 2 (Multiple Colliding Pairs, 35 points). Suppose that \( n \) pigeons are placed into \( m \) holes with \( n \geq m - 1 + 2k \) for some positive integer \( k \). Show that one can find \( k \) disjoint pairs of pigeons \((p_1, q_1), (p_2, q_2), \ldots, (p_k, q_k)\) so that \( p_i \) and \( q_i \) were placed in the same hole for each \( i \). Hint: use induction on \( k \).
Question 3 (Restricted Compositions, 35 points). In how many ways can one select integers $a_1, a_2, a_3, a_4$ with $1 \leq a_i \leq 9$ so that $a_1 + a_2 + a_3 + a_4 = 15$? You may leave your solution in terms of a closed formula and need not compute the numerical answer.