This homework is due on gradescope Friday May 21st at 11:59pm pacific time. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in \LaTeX is recommend though not required.

**Question 1** (Generating Function Formulas, 20 points). *Give explicit formulas for each of the following generating functions:*

(a) \[ \sum_{n=0}^{\infty} n^2 2^n x^n. \] [5 points]

(b) \[ \sum_{n=0}^{\infty} a_n x^n \text{ where } a_0 = a_1 = 1 \text{ and } a_{n+2} = 3a_{n+1} - a_n \text{ for } n \geq 0. \] [5 points]

(c) \[ \sum_{n=3}^{\infty} 3^n x^n / n! \] [5 points]

(d) \[ \sum_{n \text{ even } n \geq 0} n^2 x^n / n! \] [5 points]

**Question 2** (Generating Function Coefficients, 20 points). *Give explicit formulas for the coefficients of each of the following generating functions:*

(a) *The ordinary generating function* \[ \log \left( \frac{1}{1-x} \right) / (1-x) \] *.[5 points]*

(b) *The ordinary generating function* \[ 1 / (x^2 - x - 6) \]. *.[5 points]*

(c) *What recurrence relation defines the coefficients of the ordinary generating function* \[ 1 / (x^3 + 2x + 1) \] ? *.[5 points]*

(d) *The exponential generating function* \[ e^x \sin(x) \]. *.[5 points]*

**Question 3** (Combining Generating Functions, 60 points). *Give generating functions whose coefficients give the answers to the following combinatorial problems:*

(a) *[15 points] Let* \( c_{n,m} \) *be the number of compositions whose numbers are colored red and blue where the sum of the red numbers is* \( n \) *and the sum of the blue numbers is* \( m \). *Give a formula for the generating functions\[ \sum_{n,m=0}^{\infty} c_{n,m} x^n y^m. \] *

(b) *[15 points] Let* \( p_d(n) \) *be the number of partitions of* \( n \) *where no part has size a multiple of 3 and no two parts have sizes differing by 0 or 1. *Give a formula for\[ \sum_{n=0}^{\infty} p_d(n) x^n. \]
(c) [15 points] Let $a_n$ be the number of partitions of $[n]$ into sets $A, B, C$ where the elements of $A$ are painted red and blue, $|B|$ is odd and one element of $C$ is painted green. Give a formula for

$$\sum_{n=0}^{\infty} a_n x^n / n!$$

(d) [15 points] Let $r(n, m)$ be the number of permutations of $[n]$ with exactly $m$ cycles of length 3. Give a formula for the generating function

$$\sum_{n,m=0}^{\infty} r(n, m) (x^n / n!) y^m.$$ 

**Question 4** (Extra credit, 1 point). *Approximately how much time did you spend working on this homework?*