Math 154 Homework 3  

Spring 2020

This homework is due on gradescope by Sunday April 26th at 11:59pm pacific time. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in \LaTeX is recommend though not required.

Please cite any other students with whom you collaborated on any problems.

**Question 1** (Caley’s Theorem and Degree Sequences, 30 points). *Recall that our proof of Caley’s Theorem associated to each tree with vertices labelled $1,2,\ldots,n$ as list of $n-2$ numbers from 1 to n. Show that if a tree $T$ corresponds to a list of numbers $L$, that for every $1 \leq k \leq n$, the degree of vertex $k$ in $T$ is equal to one plus the number of times that $k$ appears in the list $L$.*

**Question 2** (Eulerian Directed Graphs, 40 points). *We only consider a directed graph to be Eulerian if the circuit that uses each edge exactly once only follows edges in the specified directions (thinking of a directed edge as a 1-way road). The condition for when a directed graph is Eulerian is now somewhat different. Prove that a connected, directed graph $G$ has an Eulerian circuit if and only if for every vertex $v$ of $G$, $d_{in}(v) = d_{out}(v)$, where $d_{in}(v)$ is the number of edges pointing into $v$ and $d_{out}(v)$ is the number of edges pointing out of $v$.*

**Question 3** (4-Regular non-Hamiltonian Graphs, 30 points). *Give an example of a connected, 4-regular, non-Hamiltonian graph. Prove that it is not Hamiltonian.*

**Question 4** (Extra credit, 1 point). *Approximately how much time did you spend on this homework?*