CSE 101 Homework 5

Spring 2018

This homework is due on gradescope Friday May 25th at 11:59pm. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in LaTeX is recommended though not required.

**Question 1** (Counting Restricted Growth Sequences, 30 points). A sequence has the restricted growth property if it is a sequence of positive integers $a_1, a_2, \ldots, a_n$ so that

1. $a_1 = 1$.
2. $a_{n+1} \leq \max_{1 \leq i \leq n}(a_i + 1)$.

Devise a polynomial time algorithm to compute the number of such sequences of length $n$.

**Question 2** (Investment Planning, 40 points). Susan owns a new startup company. They are making profits, but could make more if they upgraded their facilities. She has $n$ plans to upgrade her company's facilities. Each plan $i$ has an associated cost $c_i$, and an amount of yearly profit $p_i$ that her company would earn after upgrading to this plan (currently the office uses plan 1). Susan is trying to develop a plan for her company's growth over the next $m$ years. Initially, the company starts with $\$0$ in savings. Each year, Susan's company can do one of two things. Either it can earn money, adding $p_i$ to its savings if the current building plan $i$ is being used. Alternatively, she can pick some other plan $j$ and pay $c_j$ from her company's savings (this is only possible of course if she currently has at least that much in savings) in order to change to facility plan $j$ (her company will not be able to make money that year as the upgrade will be disruptive).

Design an algorithm to devise a plan for how Susan can run her company over the course of the next $m$ years in such a way as to maximize the total amount of savings that her company has at the end of that period. For full credit, your algorithm should run in time $O(nm)$ or better.

**Question 3** (Leave Planning, 30 points). Michael has some friends who organize day long hiking trips once a month. These friends are poor at planning though and will organize the hike on a random day of the month providing no advance warning. Unfortunately, in order to get time off of work, Michael needs to inform his boss of vacation days at least a day in advance (and in particular before he knows whether or not the hike will take place). This means that Michael is forced to schedule vacation days and hope that the hike will take place on them, however he can be strategic about it, for example, by waiting until the end of the month by which point the hike will have already happened or by guaranteed to occur the next day.

Michael can use a total of $k$ vacation days over the course of the next $m$ months. What is the greatest expected number of hikes that he can arrange to attend? For the purposes of this problem, you may assume that a month is exactly 30 days long.

**Question 4** (Extra credit, 1 point). Approximately how much time did you spend working on this homework?