CSE 101 Homework 2

Fall 2018

This homework is due on gradescope Friday October 26th at 11:59pm. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in \LaTeX is recommended though not required.

**Question 1** (Partial Dijkstra, 30 points). Suppose that when running Dijkstra’s algorithm we only need to be able to find the vertices within distance $d$ of the source node.

(a) Show that you can modify Dijkstra to achieve this in $O(|V_d| \log |V_d| + |E_d|)$ time where $V_d$ is the set of vertices within distance $d$ of the source and $E_d$ is the collection of edges with at least one endpoint in $V_d$. 
Note: This is practically very useful, as it means that you don’t need to analyze the entire map of North America in order to find the shortest path to your local grocery store. [15 points]

(b) Suppose that furthermore you know that all edge weights in the graph are non-negative integers. Show that the runtime can be improved to $O(|V_d| + |E_d| + d)$. [Hint: you will want to modify the priority queue being used in the algorithm using the knowledge that all keys will be integers at most $d$] [15 points]

**Question 2** (Street Race Planning, 30 points). Leonard and Amy are street racers. Their city is given by a directed graph $G$. For each edge $e$, they have average times $L(e)$ and $A(e)$ that it takes them to traverse that edge. Howard is trying to fix a race in Amy’s favor. To do so he needs to find a course for this race (which must start and end at the same place) so that Amy’s average time to complete the course is smaller than Leonard’s. Give an algorithm to determine whether or not this is possible to do. For full credit, your runtime should be $O(|V||E|)$ or better.

**Question 3** (Number of Shortest Paths, 40 points). Let $G$ be a directed graph with positive edge weights and two specified vertices $s$ and $t$.

(a) Find a way to compute the set $H$ of directed edges of $G$ that are part of some shortest path from $s$ to $t$. [Note: The solution to Q2 on Exam 1 from Spring 2018 might be useful here] [10 points]

(b) Given the above, find an algorithm to compute some particular shortest path from $s$ to $t$ in $G$. [Hint: show that any path from $s$ to $t$ using only edges of $H$ is a shortest path] [10 points]

(c) Find an algorithm to compute the number of shortest paths from $s$ to $t$ in $G$. [Hint: you may want to show that $H$ is a DAG] [20 points]

For full credit, all algorithms here should be near linear (up to log factors) time.

**Question 4** (Extra credit, 1 point). Approximately how much time did you spend on this homework?