CSE 101 Homework 2

Fall 2019

This homework is due on gradescope Friday October 25th at 11:59pm on gradescope. Remember to justify
your work even if the problem does not explicitly say so. Writing your solutions in \LaTeX is recommend
though not required.

Question 1 (Return to Graphania, 40 points). Sylvester is still trying to plan his trip to Graphania and
their newly uni-directional road system. He is given a list of cities and a list of one-way roads connecting
pairs of them (i.e. a directed graph G on the cities). For each city he is given the price of the cheapest
flight from his home to that city, and the cheapest flight from that city back home (there are no plane flights
between cities in Graphania). He is trying to find the cheapest trip consisting of flying to some city from
home, driving along some sequence of roads (free), and then flying home from the resulting city. For each of
the following, provide a linear time algorithm for computing the best possible price.

(a) Give an algorithm that works if G consists of only a single strongly connected component. [10 points]

(b) Give an algorithm that works if G is a DAG. [10 points]

(c) Give an algorithm that works for an arbitrary graph G. Hint: compute the metagraph and find a way to
combine the above ideas. [20 points]

Question 2 (Modified Priority Queue, 10 points). A Van Emde Boas priority queue is an implementation
of the priority queue data structure so that if the entries in the queue are all integers between 1 and M, the
basic operations can each be performed in \(O(\log \log(M))\) time. Suppose that Dijkstra is run on a graph G
whose edge weights are all positive integers of size at most n. What runtime would you expect if the priority
queue is implemented by a Van Emde Boas priority queue? When is this favorable to using a binary heap?
What about a Fibonacci heap?

Question 3 (Currency Trading, 50 points). Gordon works in currency trading. He has a portfolio of various
currencies. He also knows a number of vendors each willing to trade one specific currency for another specific
one at a given rate. For example, he might know someone willing to trade him \(1.1\) US Dollars for a Euro.

(a) Gordon wants to rebalance his portfolio, trading some of his currency A for currency B. However, rather
than finding a single vendor willing to perform the transaction, he is willing to make a series of trades
in order to accomplish this. For example, he might find a vendor to exchange some of his currency A for
currency X, and another to trade that for currency Y before finally trading for currency B. He would
like to find a sequence of such trades that gives him the best possible exchange rate from A to B. Give
an algorithm that finds this best rate. You can assume that you are given a list of currencies and a list
of vendors and the trades they are willing to make at what rates. For full credit your algorithm should
run in polynomial time. [25 points]

(b) Gordon wonders if there is a way to make a profit just by trading. For example, there might be some
series of trades he could make starting with currency A that would end up with him ending up with more
of currency A than he started with. Give an algorithm to determine whether or not this is possible. For
full credit your solution should be polynomial time. [25 points]

Question 4 (Extra credit, 1 point). Approximately how much time did you spend working on this homework?