CSE 101 Homework 1

Spring 2018

This homework is due on gradescope Friday April 13th at 11:59pm. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in \LaTeX is recommended though not required.

**Question 1** (Gravity Maze, 30 points). Harry is trying to navigate a three dimensional maze. The maze is constructed using an \( n \times n \times n \) grid of cubes where each cube is either open space that can be traversed or a wall which cannot. Harry’s location at any point in time must be one of the open cubes. Harry cannot climb, but he does have the ability to manipulate gravity. At any point gravity must be pointing in one of the six cardinal directions (up, down, north, south, east, or west, these are aligned with the axes of the cube). Harry can do two things to traverse the maze. Firstly, he can take a step in one of the six cardinal directions to an adjacent open cube in the maze, and then fall as many squares in the current direction of gravity as possible until a wall or the end of the maze stops him. His other option is that he can change the direction that gravity points 90 degrees to a different one of the cardinal directions.

Produce an algorithm that given a description of the maze (which squares are open, and which are walls), the location of the exit, Harry’s initial location, and the initial direction of gravity determines whether or not it is possible for Harry to complete the maze. The runtime for this algorithm should be polynomial in \( n \).

**Question 2** (Best Neighborhood, 30 points). Murphy is looking to buy a house. One of her major considerations is the ability to walk to places that she might want to go. The city she lives in is described by an undirected graph \( G \) whose vertices are locations and whose edges denote walking paths. Each location is assigned by Murphy a non-negative number of points. Murphy wants to find a location so that the greatest possible total number of points of other locations are reachable on foot from there.

(a) Give a linear time algorithm for solving this problem. [25 points]

(b) Does the above algorithm work if \( G \) were a directed graph instead? Why or why not? [5 points]

**Question 3** (Preorder Numbers and Graph Structure, 40 points). We are given a graph \( G \) on \( n \) vertices. Depth first search is run on \( G \) and pre- and post-order numbers are computed.

(a) If \( G \) is an undirected graph, and the pre-order numbers are \( 1, 2, 4, 6, \ldots, 2n - 2 \), what can you say about the structure of \( G \)? [10 points]

(b) If \( G \) is a directed graph, and the pre-order numbers are \( 1, 2, 3, \ldots, n \), what can you say about \( G \)? [10 points]

(c) If \( G \) is an undirected graph, and the pre-order numbers are \( 1, 3, 5, 7, \ldots, 2n - 1 \), what can you say about \( G \)? [10 points]

(d) If \( G \) is a directed graph, and the pre-order numbers are \( 1, 3, 5, 7, \ldots, 2n - 1 \), show that \( G \) is a DAG. [10 points]

**Question 4** (Extra credit, 1 point). Approximately how much time did you spend working on this homework?