CSE 101 Homework 1

Fall 2019

This homework is due on gradescope Friday October 11th at 11:59pm on gradescope. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in \LaTeX is recommend though not required.

**Question 1** (Bottle Game, 35 points). You want to collect exactly $t$ ounces of water. To accomplish this, you have four bottles which store $n_1, n_2, n_3, n_4$ ounces of water respectively for some positive integers $n_i$. You can attempt to reach your goal by applying some sequence of the following operations:

- Filling a bottle from the sink.
- Emptying a bottle onto the ground.
- Pouring one bottle into another until either the first empties or the second fills.

All four bottles start empty. Give an algorithm to determine whether or not it is possible to apply some sequence of the above operations in order to end up with some bottle containing exactly $t$ ounces of water. For full credit, your algorithm should run in time $O((n_1 + n_2 + n_3 + n_4)^3)$ or better.

**Question 2** (Vacation Planning, 35 points). Sylvester is planning a trip to Graphania. He has a road map, showing how cities are connected by (two-way) roads. He also has a list of prices for the cheapest flight flying into each city and the cheapest flight flying out of each city. He wants to plan a trip whereby he flies into one city, drives to another city (assume that this costs nothing) and flies out for as little total money as possible.

(a) Give a linear time algorithm for solving this problem. [25 points]

(b) Rock slides close some of the lanes of some roads, turning them into one-way roads. Does the above algorithm still work? Why or why not? [10 points]

**Question 3** (Cycles and Orderings, 30 points). Let $G$ be a directed graph and $C$ a cycle of its vertices. Assume that we run DFS on $G$ computing pre- and post-order numbers. For the following statements, either prove them or provide a counterexample.

(a) The vertex of $C$ with the smallest preorder number has the largest postorder number. [15 points]

(b) The vertex of $C$ with the largest preorder number has the smallest postorder number. [15 points]

**Question 4** (Extra credit, 1 point). Approximately how much time did you spend working on this homework?