CSE 101 Final Exam

Spring 2018

Instructions: Do not open until the exam starts. The exam will run for 180 minutes. The problems are roughly sorted in increasing order of difficulty. Answer all questions completely. You are free to make use of any result in the textbook or proved in class. You may use up to 12 1-sided pages of notes, and may not use the textbook nor any electronic aids. Write your solutions in the space provided, the pages at the end of this handout, or on the scratch paper provided (be sure to label it with your name). If you have solutions written anywhere other than the provided space be sure to indicate where they are to be found. Please sit in the seat indicated below.

If the problem asks for an algorithm, giving a correct algorithm with worse runtime efficiency than what is asked for will be awarded partial credit.

Name:

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**Question 1** (Huffman Code, 15 points). Consider the following set of letters each occurring with the associated frequencies: A - 2, B - 3, C - 7, D - 8, E - 10, F - 15, G - 20, H - 40, I - 50. Give the tree for the corresponding Huffman code. [It is suggested that you show your work so that it will be easier to assign partial credit if your answer is wrong.]
Question 2 (Strongly Connected Components, 15 points). What are the strongly connected components of the graph below?

[It is suggested that you show your work so that it will be easier to assign partial credit if your answer is wrong.]
Question 3 (Distance Approximation, 15 points). Given a set of integers $A$ of size $n$ and an integer $T$, give an algorithm to find the smallest possible value of $|x - y - T|$ over all pairs $x, y \in A$. For full credit your algorithm should run in time $O(n \log(n))$ or better.
**Question 4** (Board Game Probability, 15 points). *Johnny is playing a board game. His piece is on a track and on every turn he rolls a fair six sided die and advances his piece that many squares. Johnny is attempting to reach a square \( n \) steps ahead of his current location. Give an algorithm to compute the probability that he will reach or pass this square within the next \( k \) turns and analyze the runtime of this algorithm. For full credit your algorithm should have runtime \( O(nk) \) or better.*
**Question 5** (Shortest Simple Path, 20 points). Consider the following problem **ShortestSimplePath**, which given a weighted graph $G$ (possibly with negative edge weights), and vertices $s,t$ of $G$ asks for the smallest weight of a path in $G$ from $s$ to $t$ that uses no vertex more than once. Show that **ShortestSimplePath** is NP-Hard.
Question 6 (Closest Pair of Points, 20 points). Given an unweighted, undirected graph $G$ and a set $S$ of the vertices in $G$ give an algorithm to compute the length of the shortest path connecting two different vertices of $S$. For full credit, your algorithm should run in linear time.