CSE 101 Exam 2

Fall 2019

Instructions: Do not open until the exam starts. The exam will run for 45 minutes. The problems are roughly sorted in increasing order of difficulty. Answer all questions completely (though pay attention to exactly what the question is asking for). You are free to make use of any result in the textbook or proved in class. You may use up to 6 1-sided pages of notes, and may not use the textbook nor any electronic aids. Write your solutions in the space provided, the pages at the end of this handout, or on the scratch paper provided (be sure to label it with your name). If you have solutions written anywhere other than the provided space be sure to indicate where they are to be found.

If the problem asks for an algorithm, giving a correct algorithm with worse runtime efficiency than what is asked for will be awarded partial credit.

Name:

ID Number:

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Question 1 (Shortest Path, 30 points). *Give the shortest path from S to T in the following graph (give the path):*
Question 2 (Best Leaf, 35 points). You are given a balanced binary tree \( T \) with \( n \) leaves. Each leaf is labelled with a real number, and each other vertex is labelled with an operation either \(+a\) or \(\times b\) for \(a\) or \(b\) a positive real number. You want to pick a leaf and apply the relevant operations one at a time going up the tree. You would like to find the leaf that maximizes this value. For example in the example below, the selected leaf gives the largest value of \((5 \times 2) + 1 = 11\).

Give an algorithm to compute the largest possible value achievable this way. For full credit your algorithm should run in time \(O(n)\) or better.
Question 3 (Farm Work, 35 points). Lillian is a farmhand and is trying to plan her work for the season. She has a map (an undirected graph) of the country in which she works along with transportation costs for the various roads. She needs to plan work for both apple picking season and banana picking season. At each city (vertex) she knows the pay of the best apple picking job at that location and the pay of the best banana picking job. She wishes to plan a route from her home to an apple picking job, then to a banana picking job (possibly in the same city) and then back home in such a way as to maximize her income minus expenses. Give an algorithm to compute the best possible net profit. For full credit, your algorithm should run in near-linear time.