CSE 101 Homework 3

Winter 2015

This homework is due Friday February 13th at the start of class. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in \LaTeX is recommended though not required.

Question 1 (Deterministic Order Statistics, 40 points). Although the algorithm that we saw in class for computing order statistics requires randomization, it turns out that this is not required to find an efficient algorithm. Consider the following algorithm for order statistics:

\textbf{DeterministicOrderStatistics}(L,k) \ finds the kth largest element in list L

Split the elements of L into groups of 5 elements each

Compute the medians of each group of 5 (using some constant time algorithm for each one)

Put these medians together in a list M

Let x = \textbf{DeterministicOrderStatistics}(M,|M|/2) \ x is the median of M

Compare all elements of L to x and sort them into lists S (the elements smaller than x) and B (the elements larger than x)

If |B| > k-1 \ use x as a pivot as before to reduce to a problem of smaller size

Return \textbf{DeterministicOrderStatistics}(B,k)

If |B| = k-1

Return x

If |B| < k-1

Return \textbf{DeterministicOrderStatistics}(S,k-1-|B|)

(a) Show that if |L| = n that this algorithm makes one recursive call on a list size at most $n/5 + O(1)$ and one on a list of size at most $7n/10 + O(1)$. [15 points]

(b) Write a recurrence relation for the runtime of this algorithm on lists of size n. [10 points]

(c) Prove by induction that the runtime of this algorithm is $O(|L|)$. Note that you will need to be careful to show that constant in your big-O does not change between inductive steps (recall the last problem of Homework 0). [15 points]

Question 2 (Echoes and Convolutions, 35 points). David is exploring a cave. He finds that when he shouts at volume $V_i$ exactly $i$ seconds later for each $1 \leq i \leq n$ (though perhaps some of the $a_i$ are 0, meaning that there is no echo at that time). In fact, if he considers the initial shout to be a trivial echo with $a_0 = 1$, all of the noise produced in this way can be thought of as 'echoes'. David also discovered that if he shouts several times that the volumes of the various echoes of his shouts add together.

(a) Suppose that at some starting time, $t$, David shouts at volume $V_i$ exactly $i$ seconds after $t$ for each $1 \leq i \leq n$. Give a formula to express the total volume $R_i$ of all echoes that David hears $i$ seconds after the start time. [10 points]

(b) Give an $O(n \log(n))$ algorithm to compute the values of the $R_i$ given the $a_i$ and the $V_i$. [10 points]

(c) Suppose that Eve is travelling with David. By analyzing the caves, she is able to determine the values of the $a_i$, and by listening to echoes can determine the values of the $R_i$. Give an $O(n \log(n))$ time algorithm to show how she could reconstruct the values of the $V_i$ from this data. Hint: if convolution corresponds to polynomial multiplication, how would we undo this operation? [15 points]
Question 3 (Binary Search On Unimodal Lists, 25 points). Call a list $L$ of numbers unimodal if the elements of the list increase for a while and then decrease. In particular, a list $L$ of length $n$ is unimodal if $L[1] < L[2] < L[3] < \ldots < L[k] > L[k+1] > \ldots > L[n]$ for some index $k$.

(a) Give an $O(\log(n))$ time algorithm to find the index of the largest element of a unimodal list of length $n$ (so in the example above, your algorithm should return $k$). [10 points]

(b) Give an $O(\log(n))$ time algorithm to search a unimodal list for a given element. In particular, given a unimodal list $L$ of length $n$ and a number $x$ your algorithm should determine whether or not $x$ is an element of $L$. [15 points]

Question 4 (Extra credit, 1 point). Approximately how much time did you spend working on this homework?