CSE 101 Homework 6

Spring 2016

This homework is due Friday June 3rd, 11pm on gradescope. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in \LaTeX is recommend though not required.

**Question 1** (Mastermind, 60 points). Mastermind is a common two-player logic game. One player, the codemaker, creates a secret sequence of colors $c_1, \ldots, c_n$. The other player, the codebreaker, makes a sequence of guesses each of which is another sequence $b_1, \ldots, b_n$ of the same length. For each guess, the codemaker tells the codebreaker both the number of colors that are correct and in the correct position (i.e. the number of indices $i$ so that $c_i = b_i$), and additionally the number of colors that were correct but in the incorrect location. In particular, this is the sum over all colors $C$ of the minimum of the number of mismatched $c_i$ of color $C$ and the number of mismatched $b_i$ of color $C$. For example if the secret sequence is RRBG and the guessed sequence is RBRR, this has one in the correct position and color (the first R) and two additional that are the correct color in the wrong position (one of the other R’s and the B). The third R in the guessed sequence is not counted because there is not a corresponding third R in the secret sequence.

The Mastermind-Consistency-Problem is the computational problem defined as follows. You are given a number of guesses and for each guess the numbers of correct colors in correct positions and correct colors in incorrect positions. Your goal is to determine whether or not there is a secret sequence that is consistent with this information.

(a) Show that this problem is in NP. [20 points]

(b) Show that this problem is NP-complete. [40 points] Hint: There is a relatively easy reduction from Zero-One Equations. Construct a scenario where the $i$th color $c_i$ is either Red or Blue. Use this to encode the value of the variable $x_i$ in the Zero-One Equation. Use the guesses to encode the equations of the Zero-One Equation.

**Question 2** (Backtracking and 3-SAT, 40 points). Suppose that you are given a 3-SAT problem with $n$ variables. Note that the obvious algorithm of trying all possible settings of the variables takes time roughly $2^n$. Show that there is an algorithm for 3-SAT on $n$ variables that runs in time $O(C^n)$ for some constant $C < 2$. Hint: Consider using a backtracking algorithm. In particular, if you simultaneously assign values to all three variables in a clause, you have used up three variables, but there are only 7 possible ways to assign values to these variables rather than 8.

**Question 3** (Extra credit, 1 point). Free point! Please remember to fill out your CAPEs for this course.