CSE 101 Homework 1

Spring 2016

This homework is due Friday April 8st, 11pm on gradescope. Remember to justify your work even if the problem does not explicitly say so. Writing your solutions in \LaTeX is recommend though not required.

Question 1 (Airport placement, 35 points). The nation of Graphia, has a number of cities, some pairs of which are connected by roads (all of which are two-way). Unfortunately, due to various hazardous terrain, not all cities are reachable from all other cities. There is a plan to fix this problem by building airports in some number of the cities. The requirement is that it must be possible to reach a least one airport from any starting city. Furthermore, each city has an airport cost which is a positive integer denoting the cost of building an airport in that city.

(a) Find a linear time algorithm that given the airport costs and the connections made by the roads, returns the cheapest collection of cities to build airports in so that every city will be reachable from at least one airport. Show that your algorithm is correct and runs in an appropriate amount of time. [30 points]

(b) What goes wrong with this algorithm if some of the roads are one-way? [5 points]
**Question 2** (Puzzle Solving, 35 points). John is trying to solve a puzzle. In this puzzle he has an $n \times n$ board. Some of the squares are marked with obstacles, and one of which is a designated target square. The board also has two pawns on it. As a move, John makes take either pawn and move it as far as it goes in one of the four cardinal directions until it bumps into an obstacle, the edge of the board, or the other pawn. His objective is to make some sequence of moves that ends with one of the pawns at the target square. For example, in the figure below, the moves shown allow him to bring pawn A to the target square in 6 moves. Find an algorithm that given $n$, and the locations of the obstacles, pawns, and target runs in time polynomial in $n$ and determines whether or not it is possible for John to solve this puzzle. What is the runtime of this algorithm?

*Hint:* Relate this question to one of reachability in some appropriate graph.
Question 3 (Largest Previsit Numbers, 30 points). Suppose that $G$ is a connected, undirected graph on $n > 1$ vertices.

(a) Show that when doing a depth first search on $G$ that the largest previsit number of any vertex is at most $2n - 2$. [5 points]

(b) Give an example of a connected graph on 5 vertices and a depth first search traversal of this graph so that some vertex has previsit number exactly 8. [10 points]

(c) Show that when doing a depth first search on $G$ that the largest previsit number is at least $n$. [5 points]

(d) Give an example of a connected graph on 5 vertices and a depth first search traversal of this graph so that the largest previsit number is exactly 5. [10 points]

Question 4 (Extra credit, 1 point). Approximately how much time did you spend working on this homework?