CSE 101 Final Exam

Spring 2016

Instructions: Do not open until the exam starts. The exam will run for 180 minutes. The problems are roughly sorted in increasing order of difficulty. Answer all questions completely. You are free to make use of any result in the textbook or proved in class. You may use up to 10 1-sided pages of notes, and may not use the textbook nor any electronic aids. Write your solutions in the space provided, the pages at the end of this handout, or on the scratch paper provided (be sure to label it with your name). If you have solutions written anywhere other than the provided space be sure to indicate where they are to be found.

For several problems on this exam an algorithm is asked for with a specific runtime. Providing a working algorithm with worse runtime will usually receive some partial credit. Some questions will also ask you for justifications of your stated runtime or of the algorithm correctness. You will not be required to provide these things unless asked explicitly, so please be sure to read the problem statements carefully.

Please sit in a location that is not directly next to nor directly in front of or behind any other students.

Name:

ID Number:

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Question 1 (Large Cut, 15 points). Give an explicit black and white coloring of the vertices of the graph below so that all vertices are adjacent to at least as many vertices of the opposite color as they are to vertices of the same color.
**Question 2** (Interval Packing, 15 points). Provide as large a collection of intervals from the following set so that no two chosen intervals overlap.

\[ A = [1, 4], B = [2, 8], C = [3, 5], D = [6, 7], E = [9, 16], F = [10, 20], G = [11, 14], H = [12, 19], I = [13, 17], J = [15, 18]. \]
Question 3 (Subset Sum Reduction, 15 points). Consider the following two slightly different versions of the subset sum problem.

In the targeted subset sum problem, you are given a set $S = \{a_1, \ldots, a_n\}$ of positive integers and an integer $C$. The objective is to determine whether or not there is a subset of $S$ whose elements sum to $C$.

In the zero subset sum problem, you are given a set $S = \{a_1, \ldots, a_n\}$ of integers. The objective is to determine whether or not there is a nonempty subset of $S$ whose elements sum to 0.

Find a reduction from the targeted subset sum problem to the zero subset sum problem and prove that it is correct.
**Question 4** (Pareto Optimality, 15 points). Given a set $S = \{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\}$ of elements of $\mathbb{Z}^2$, say that a point $(x_i, y_i)$ is Pareto optimal as long as there is no $j \neq i$ so that $x_j \geq x_i$ and $y_j \geq y_i$. For example, if $S = \{(1, 3), (2, 4), (3, 1)\}$, $(2, 4)$ and $(3, 1)$ are Pareto optimal but $(1, 3)$ is not.

Give an algorithm that given $S$ returns the set of Pareto optimal points in $S$, and analyze its runtime. For full credit, your algorithm should run in $O(n \log(n))$ time.
**Question 5** (k-Interval Cover, 20 points). Define the k-interval cover problem as follows. Given a set $S = \{x_1, x_2, \ldots, x_n\}$ of integers, and a positive integer $k$, the objective is to find $k$ intervals $I_1, I_2, \ldots, I_k$ so that each $x_i$ is in one of the intervals $I_j$ and so that the sum of the squared lengths of the intervals $I_k$ is as small as possible. For example, if $S = \{1, 3, 6\}$ and $k = 2$, the best solution is $I_1 = [1, 3], I_2 = [6, 6]$, and the sum of squares of the lengths is 4.

Give an algorithm that given $S$ and $k$ returns the sum of the squared lengths of the intervals in a solution to the k-interval cover problem. Prove that your algorithm is correct. For full credit, your algorithm should run in time $O(kn^2)$ or better.
Question 6 (Shopping Trip Planning, 20 points). Jade lives in a town whose streets (all of which are the same length) are given by the edges of an undirected graph $G$. Her home is located at a specified vertex $H$. Jade wants to go on a shopping trip where she visits each of $k$ different stores in order. However, each store may have more than one location in her town. In particular, for each $1 \leq i \leq k$ there is a set $S_i$ of vertices at which a branch of the $i$th store is located (you may assume that the $S_i$ are disjoint).

Give an algorithm that given $G$, $H$, and the $S_i$ gives the length of the shortest path that starts and ends at $H$ and passes through a vertex of each of the $S_i$ in order. In other words, it starts at $H$, then passes through a vertex of $S_1$, then a vertex of $S_2$, then a vertex of $S_3$, and so on eventually passing through a vertex of $S_k$ and finally back to $H$. It may pass through some of the other marked vertices in the meantime. For example, in the example below the path of length 10 shown is the shortest one that visits $H, 1, 2, 3, H$ in order.

For full credit, your algorithm should run in time $O(k(|V| + |E|))$ or better.