

CSE 207 — Modern Cryptography

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Website: <http://www-cse.ucsd.edu/users/mihir/cse207>

Did you use any cryptography

- today?

Did you use any cryptography

- today?
- over the last week?

Did you use any cryptography

- today?
- over the last week?
- over the Christmas break?

Cryptography usage



Ordering from Amazon.com is quick and easy

Enter your e-mail address:

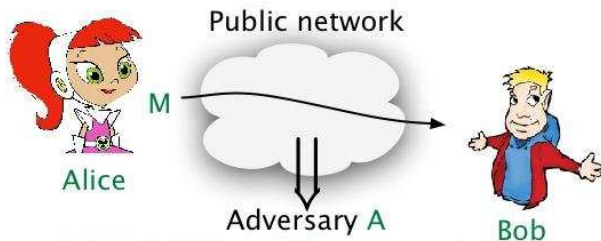
I am a new customer.
(You'll create a password later)

- https invokes the Secure Socket Layer (SSL) communication security protocol to securely transmit your credit card number to the server
- SSL uses cryptography

Other uses of cryptography

- ATM machines
- On-line banking
- Remote login and file transfer using SSH

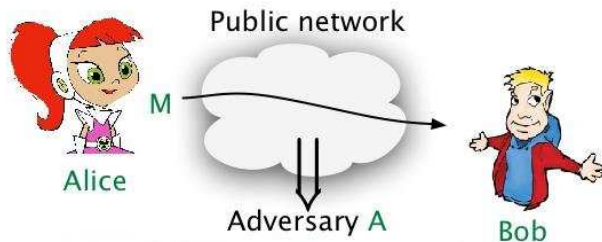
What is cryptography about?



Adversary: clever person with powerful computer

Goals:

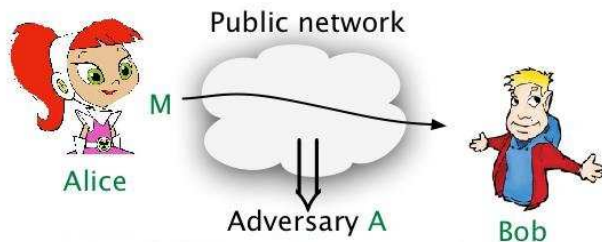
- Data privacy
- Data integrity and authenticity



The goal is to ensure that the adversary does not see or obtain the data (message) M .

Example: M could be a credit card number being sent by shopper Alice to server Bob and we want to ensure attackers don't learn it.

Integrity and authenticity



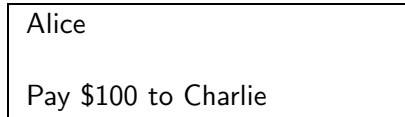
The goal is to ensure that

- M really originates with Alice and not someone else
- M has not been modified in transit

Integrity and authenticity example

Alice

Alice
Pay \$100 to Charlie



Bob
(Bank)

Adversary Eve might

- Modify "Charlie" to "Eve"
- Modify "\$100" to "\$1000"

Integrity prevents such attacks.

Medical databases

Doctor

Reads F_A

Modifies F_A to F'_A

Get Alice
→
 F_A
←

Put: Alice, F'_A
→

Database

Alice	F_A
Bob	F_B

Alice	F'_A
Bob	F_B

Medical databases

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Database

Alice	F_A
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- Privacy: F_A, F'_A contain confidential information and we want to ensure the adversary does not obtain them

Medical databases

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Reads F_A

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Get Alice
 $\xrightarrow{F_A}$
 $\xleftarrow{F_A}$

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 $\xrightarrow{\hspace{10em}}$

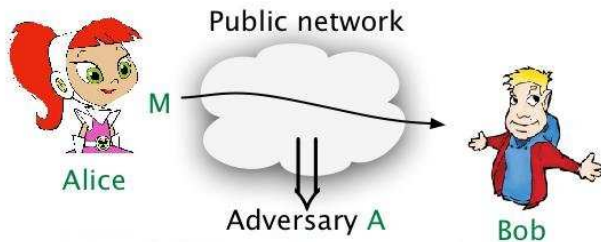
Database

Alice	F_A
Bob	F_B

Alice	F'_A
Bob	F_B

- Privacy: F_A, F'_A contain confidential information and we want to ensure the adversary does not obtain them
- Integrity and authenticity: Need to ensure
 - doctor is authorized to get Alice's file
 - F_A, F'_A are not modified in transit
 - F_A is really sent by database
 - F'_A is really sent by (authorized) doctor

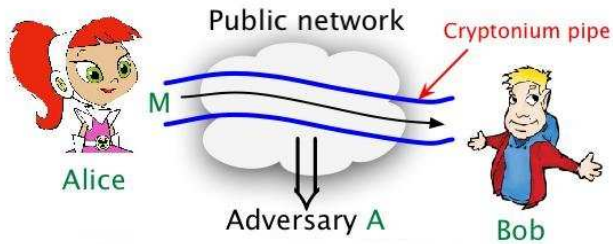
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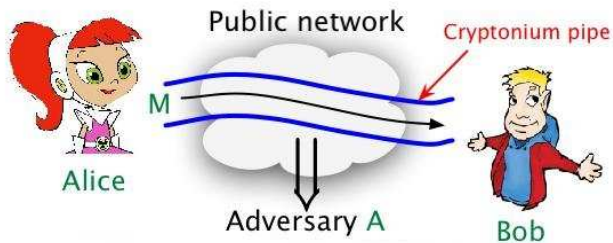
Adversary: clever person with powerful computer

Goals:

- Data privacy
- Data integrity and authenticity

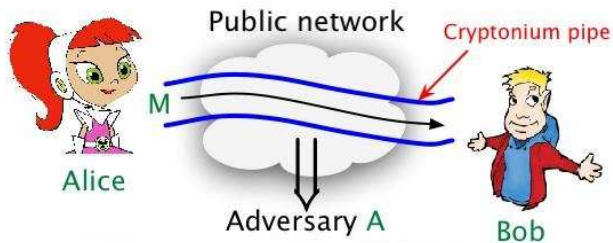


Cryptonium pipe: Cannot see inside or alter content.



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All our goals would be achieved!

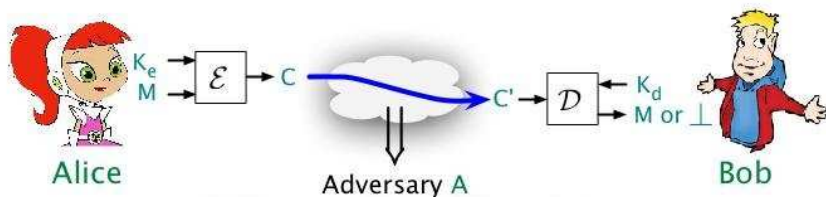


Cryptonium pipe: Cannot see inside or alter content.

All our goals would be achieved!

But cryptonium is only available on **planet Crypton** and is in **short supply**. 😞

Cryptographic schemes



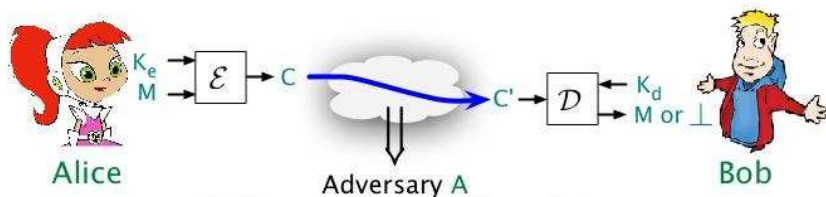
\mathcal{E} : encryption algorithm

K_e : encryption key

\mathcal{D} : decryption algorithm

K_d : decryption key

Cryptographic schemes



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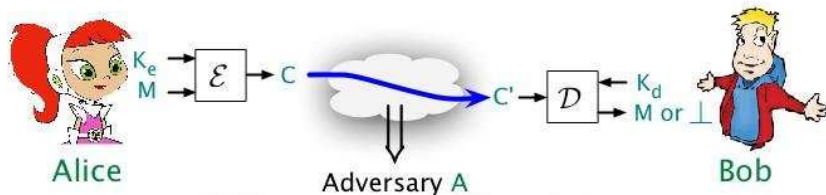
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Algorithms: standardized, implemented, public!

Cryptographic schemes



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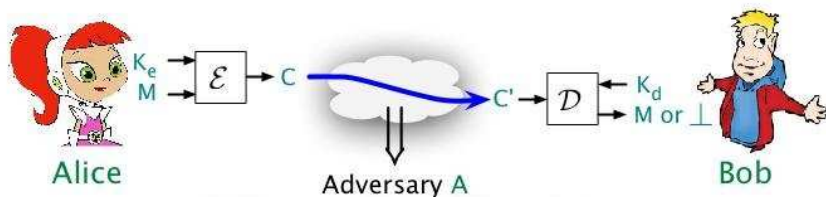
\mathcal{D} : decryption algorithm

K_d : decryption key

Settings:

- public-key (asymmetric): K_e public, K_d secret
- private-key (symmetric): $K_e = K_d$ secret

Cryptographic schemes



\mathcal{E} : encryption algorithm

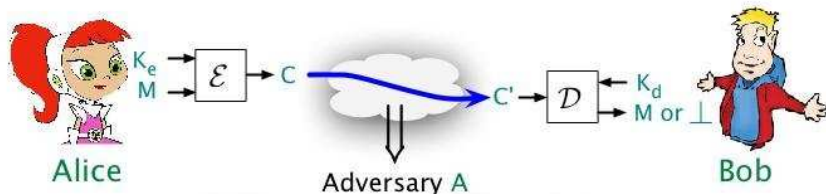
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\mathcal{D} : decryption algorithm

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How do keys get distributed? Magic, for now!

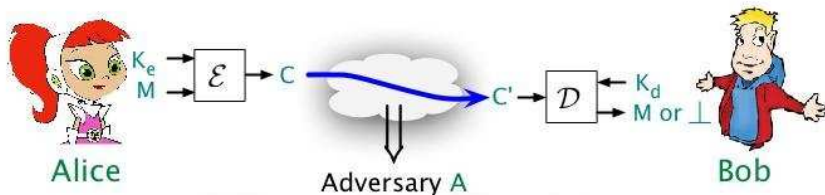
Cryptographic schemes



Our concerns:

- How to define security goals?
- How to design \mathcal{E} , \mathcal{D} ?
- How to gain confidence that \mathcal{E} , \mathcal{D} achieve our goals?

Cryptographic schemes



Computer Security: How does the computer/system protect K_e/K_d from break-in (viruses, worms, OS holes, ...)? (CSE 127,227)

Cryptography: How do we use K_e, K_d to ensure security of communication over an insecure network? (CSE 107,207)

Why is cryptography hard?

- One **cannot anticipate** an adversary strategy in advance; number of possibilities is **infinite**.
- “**Testing**” is not possible in this setting.

Substitution ciphers/Caesar ciphers:

$K_e = K_d = \pi: \Sigma \rightarrow \Sigma$, a secret permutation

e.g., $\Sigma = \{A, B, C, \dots\}$ and π is as follows:

σ	A	B	C	D	...
$\pi(\sigma)$	E	A	Z	U	...

$$\begin{aligned}\mathcal{E}_\pi(CAB) &= \pi(C)\pi(A)\pi(B) \\ &= ZEA\end{aligned}$$

$$\begin{aligned}\mathcal{D}_\pi(ZEA) &= \pi^{-1}(Z)\pi^{-1}(E)\pi^{-1}(A) \\ &= CAB\end{aligned}$$

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Not very secure! (Common newspaper puzzle)

The age of machines

Enigma: German World War II machine



Broken by British in an effort led by Turing

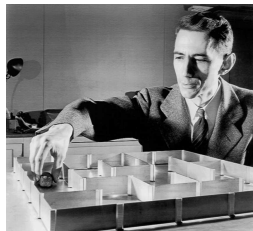
Shannon and One-Time-Pad (OTP) Encryption

$$K_e = K_d = \underbrace{K \stackrel{\$}{\leftarrow} \{0,1\}^k}_{\substack{K \text{ chosen at random} \\ \text{from } \{0,1\}^k}}$$

For any $M \in \{0,1\}^k$

- $\mathcal{E}_K(M) = K \oplus M$

- $\mathcal{D}_K(C) = K \oplus C$

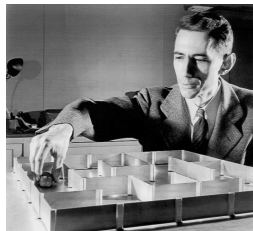


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Theorem (Shannon): OTP is perfectly secure as long as only one message encrypted.

“Perfect” secrecy, a notion Shannon defines, captures mathematical impossibility of breaking an encryption scheme.

Fact: if $|M| > |K|$, then **no scheme is perfectly secure**.

Security of a “practical” system must rely not on the impossibility but on the computational difficulty of breaking the system.

(“Practical” = more message bits than key bits)

Rather than:

"It is impossible to break the scheme"

We might be able to say:

"No attack using $\leq 2^{160}$ time succeeds with probability $\geq 2^{-20}$ "

I.e., Attacks can exist as long as **cost to mount them** is **prohibitive**, where
Cost = computing time/memory, \$\$\$

Security of a “practical” system must rely not on the impossibility but on the computational difficulty of breaking the system.

Cryptography is now not just mathematics; it needs to draw on computer science

- Computational complexity theory (CSE 105,200)
- Algorithm design (CSE 101,202)

Scheme 1.1

Classical Approach: Iterated design

Scheme 1.1 → bug!

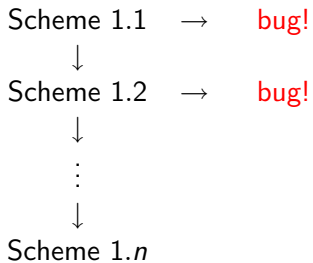
Classical Approach: Iterated design

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↓
Scheme 1.2

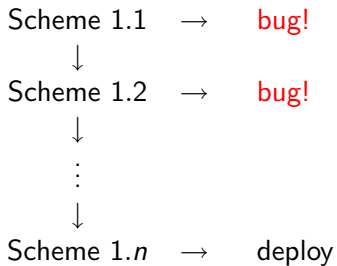
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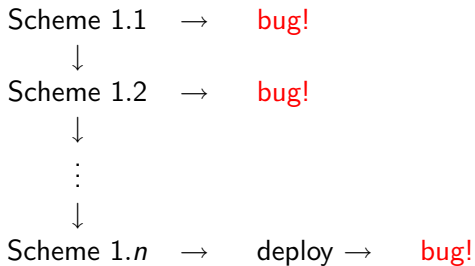
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Classical Approach: Iterated design



Classical Approach: Iterated design



- Understanding the goals: Formal **adversarial models** and **definitions of security goals**
- Beyond iterated design: Proof by **reduction** that a construction achieves its goal

A great deal of design tries to produce schemes without first asking:

“What exactly is the security goal?”

This leads to schemes that are complex, unclear, and wrong.

Being able to precisely state what is the security goal of a design is challenging but important.

We will spend a lot of time developing and justifying strong, precise notions of security.

Thinking in terms of these precise goals and understanding the need for them may be the most important thing you get from this course!

The factoring problem

Input: Composite integer N

Desired output: prime factors of N

Example:

Input: 85

Output:

The factoring problem

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Can we write a factoring program? Easy!

Alg Factor(N) // N a product of 2 primes

For $i = 2, 3, \dots, \lceil \sqrt{N} \rceil$ do

 If $N \bmod i = 0$ then return i

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But this is very slow ...

Prohibitive if N is large (e.g., 400 digits)

Can we factor fast?

- Gauss couldn't figure out how
- Nor does anyone know now



Nobody today knows how to factor a 400 digit number in a practical amount of time.

Provide

- A scheme
- A proof of security

The proof establishes something like:

“The only way to break the scheme is to factor a large number”

or, put another way

“If an adversary breaks the scheme, it must have found a fast factoring algorithm.”

Bug in scheme implies

- attacker has found a way to factor fast
- attacker is smarter than Gauss
- and smarter than all living mathematicians...

Atomic Primitives or Problems

Examples:

- **Factoring:** Given large $N = pq$, find p, q
- **Block cipher primitives:** DES, AES, ...
- **Hash functions:** MD5, SHA1, ...

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Features:

- Few such primitives
- Bugs rare
- Design an **art**, confidence by **history**.

Atomic Primitives or Problems

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- **Block cipher primitives:** DES, AES, ...
- **Hash functions:** MD5, SHA1, ...

Features:

- Few such primitives
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- Design an **art**, confidence by **history**.

Drawback: Don't **directly** solve any security problem.

Higher Level Primitives

Goal: Solve security problem of **direct** interest.

Examples: encryption, authentication, digital signatures, key distribution, ...

Higher Level Primitives

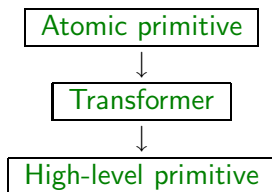
Goal: Solve security problem of **direct** interest.

Examples: encryption, authentication, digital signatures, key distribution, ...

Features:

- **Lots of them**
- **Bugs common** in practice

We typically design high-level primitives from atomic ones



History shows that the Transformer is usually the weak link:

- Atomic primitives secure, yet
- Higher level primitive insecure

Enables us to get transformers for which we can guarantee

Atomic primitive secure \Rightarrow High-level primitive secure

I.e., If attacker breaks encryption scheme then they are smarter than Gauss.

Proven-secure schemes in use (SSL, SSH, IPSec, ...):

- HMAC
- OAEP
- ECIES
- ...

Cryptography uses

- Number theory
- Combinatorics
- Modern algebra
- Probability theory

Modern Cryptography: Esoteric mathematics?

Hardy, in his essay [A Mathematician's Apology](#) writes:

“Both Gauss and lesser mathematicians may be justified in rejoicing that there is one such science [number theory] at any rate, and that their own, whose very remoteness from ordinary human activities should keep it gentle and clean”



No longer: Number theory is the basis of modern public-key systems such as RSA.

Cryptography beyond communication security

Parties $1, 2, 3, \dots, n$.

Party i has the integer $x_i \in \{0, \dots, M - 1\}$

They want to know

$$x = \frac{x_1 + \dots + x_n}{n}$$

but each party i wants to keep its own x_i private.

Cryptography beyond communication security

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Usage:

x_i = score of student i on homework 1

x_i = vote of party i for proposition X on ballot

⋮

Cryptography beyond communication security

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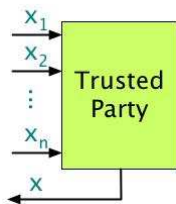
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Trusted Party Solution:



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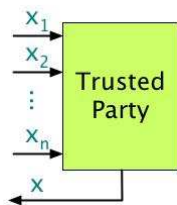
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Trusted Party Solution:



Secure Computation: Allows us to accomplish objective **without a trusted party**, using only (secure) communication between parties.

Internet Gambling



Will you play?

Internet Gambling



Will you play?

Casino can cheat. It returns 😞, T for some $T \neq g$

Internet Gambling



Will you play?

Casino can cheat. It returns ☹️, T for some $T \neq g$

Crypto can fix this!

- Millions of dollars of loss due to credit-card fraud, phishing, identity theft, ...
- Lack of privacy: Enormous amounts of information about each of us is collected and harvested by businesses dedicated to this purpose

Cryptography is a central tool in getting more security and privacy

Central uses: SSL, SSH, TLS, IPSEC, ...

- Poor exposition: Incomplete, unclear scheme specifications in documents
- Lack of precise goal formulations
- Complex, unclear or incorrect schemes

Lack of cryptographic education and skill in workforce.

What you can get from this course

You can get the ability to

- Identify threats
- Evaluate security solutions and technologies
- Design high-quality solutions
- Write clear, complete scheme specifications

If nothing else, develop a healthy sense of paranoia!

Resources:

- Lecture [slides](#)
- Course [notes](#)
- [Homework solutions](#)

No textbook.

All resources on course web page.

- Read [course information sheet](#)!
Handout today and on [course webpage](#).
- Grades based on
 - [homeworks](#)No exams, no projects.

- Each student must do their problem sets on their own
- Looking at solutions from [previous years](#) of the course or finding them on the [Internet](#) is not allowed.

- Strive for **neat**, **mathematically precise** and **well-written** solutions.
- **Type-setting** of homeworks is encouraged, but not mandated
- **Quality of exposition** will impact score.

This is a **theory course!** Largely **definitions** and **proofs**, although of applied value.

Needed: **undergraduate algorithms** and **theory of computation**, some **probability theory**, a little **calculus**, and

MATHEMATICAL MATURITY

Question: What is the cost of **multiplying** two k -bit numbers?

Question: I have a coin with probability p of HEADS. I flip it n times.

$$\Pr[\text{at least one HEADS}] =$$

Question: I have a coin with probability p of **HEADS**. I flip it n times.

$$\Pr[\text{at least one HEADS}] = pn$$

Because I flip n coins and each has probability p of being **HEADS**.

Question: I have a coin with probability p of **HEADS**. I flip it n times.

$$\Pr[\text{at least one HEADS}] = pn$$

WRONG! Why?

Say $p = \frac{1}{2}$ and $n = 3$. Then the “probability” is

$$pn = \frac{1}{2}(3) = \frac{3}{2} > 1 ??$$

Question: I have a coin with probability p of **HEADS**. I flip it n times.

$$\Pr[\text{at least one HEADS}] = pn$$

WRONG! Why?

Let H_i be the event that the i -th flip is heads.

$$\Pr[H_i] = p \text{ for all } 1 \leq i \leq n$$

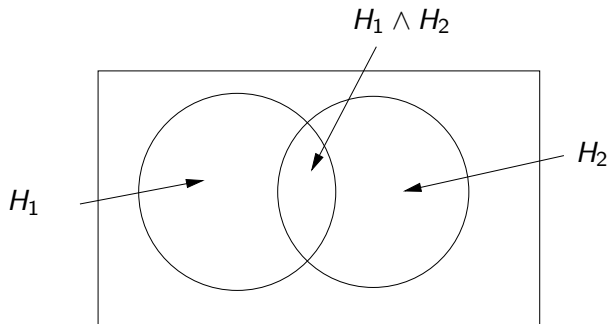
$$\Pr[\text{at least one HEADS}] = \Pr[H_1 \vee H_2 \vee \dots \vee H_n]$$

but this is not equal to

$$\Pr[H_1] + \dots + \Pr[H_n]$$

Warm-up

Example: $n = 2$



$$\Pr[H_1 \vee H_2] = \Pr[H_1] + \Pr[H_2] - \Pr[H_1 \wedge H_2]$$

Is there another way to compute

\Pr [at least one HEADs]?

Question: I have a coin with probability p of **HEADS**. I flip it n times.

$$\begin{aligned}\Pr[\text{at least one HEADS}] &= 1 - \Pr[\text{all TAILS}] \\ &= 1 - (1 - p)^n\end{aligned}$$