CSE101 - HW #1: Chapter 1, 31.(1)

Sample Solution

Case 1:

\[ g(n) = \Omega(f(n) \lg n) \]

Note that case 1 implies

\[ f(n) \lg n = O(g(n)) \]

\[ \Rightarrow \exists c, r > 0 \text{ such that } \forall n > c f(n) \lg n \leq ry(n) \]

(a) Is A faster than B for all n?

No. Let \( f(n) = 2n \) and \( g(n) = n^2 \). The final equation above is then satisfied with \( c = 1 \) and \( r = 1 \). However, for \( n < 2 \), B is faster than A and for \( n \geq 2 \) A is faster than or equal to B.

(b) Is B faster than A for all n?

No. Let \( f(n) = 2n \) and \( g(n) = n^2 \). The final equation above is then satisfied with \( c = 1 \) and \( r = 1 \). However, for \( n < 2 \), B is faster than A and for \( n \geq 2 \) A is faster than or equal to B.

(c) Is A faster than B for all n greater than some c?

Yes. By definition of \( \Omega \), \( g(n) \) grows at least as fast as \( f(n) \lg n \).

(d) Is B faster than A for all n greater than some c?

No. If B were faster than A for all n greater than some c, then

\[ \exists c > 0 \text{ such that } \forall n > c, g(n) \leq f(n) \]

This, however, is a contradiction to the above equation.