1. Two players play the following game. Player A chooses one of the three spinners below, and then player B chooses one of the two remaining spinners.

Both players then spin their spinner and the one that lands on the higher number is declared the winner. Assuming that each spinner is equally likely to land in any one of its three regions, would you rather be player A or player B? Explain your answer and justify it by computing the probabilities of the various outcomes.

2. (a) Suppose that an event A is independent of itself. Show that the probability of this event is either zero or one.

(b) Suppose that events A and B have $P(A) = 0.3$, $P(B) = 0.4$. What is $P(A \cup B)$ if A and B are independent? What is $P(A \cup B)$ if A and B are disjoint? If $P(A)$ were 0.6, and $P(B)$ were 0.8, could A and B be independent? Could they be disjoint?
3. A parallel system functions whenever at least one of its components works. Consider a parallel system consisting of $n$ independent components, each of which works with probability 0.5. Find the conditional probability that component 1 works given that the system is functioning.

4. Determine whether the following statement is true or false and prove your answer: If events $A$ and $B$ are independent, then they are conditionally independent given any event $E$.

5. Consider two independent tosses of a fair coin. Let $A$ be the event that the first toss lands heads, let $B$ be the event that the second toss lands heads, and let $C$ be the event that both land on the same side. Show that $A$, $B$, and $C$ are not independent. Show, however, that events $A$, $B$, and $C$ are pairwise independent. That is, events $A$ and $B$ are independent, events $B$ and $C$ are independent, and events $A$ and $C$ are independent.

6. (a) The probability of winning on a single roll of dice is $p$. Alice and Bob play the following game. Alice rolls the dice first, and if she fails to win, she passes the dice to Bob, who then attempts to win on his roll. They continue to pass the dice back and forth until one of them wins. What are their respective probabilities of winning?

(b) Repeat if there are $k$ players in this game, passing the dice from player $P_1$, to player $P_2$, . . . , to player $P_k$, and so on until one of them wins. What are their probabilities of winning?

(c) Repeat if there are only two players Alice and Bob, but now under the assumption that when Alice rolls the dice, she wins with probability $p_1$, while when Bob rolls the dice, he wins with probability $p_2$. What are their probabilities of winning?

7. The outcomes of an experiment are 0, 1, and 2, and they occur with probabilities $p_0$, $p_1$, and $p_2$, respectively, where $p_0 + p_1 + p_2 = 1$. Suppose that $N$ independent trials of this experiment are performed. What is the probability that outcomes 1 and 2 both occur at least once in the $N$ trials?