Let $v_1, \ldots, v_k$ be $k$ vectors in $\mathbb{R}^d$. Recall that the subspace spanned by $v_1, \ldots, v_k$ is defined as the set of all vectors $\sum_{i=1}^{k} c_i v_i$ where $c_i$'s are any scalars. The cone spanned by $v_1, \ldots, v_k$ is defined as the set of all vectors $\sum_{i=1}^{k} c_i v_i$ where the $c_i$'s are positive scalars.

Suppose we are given training data with binary $\pm 1$ labels, and we are using the Perceptron algorithm to train a classifier $\text{sign}(w^T x + b)$ on training data $(x^{(1)}, y^{(1)}), \ldots, (x^{(n)}, y^{(n)})$.

State whether following statements are true or false, and justify your answer in each case.

(1) (4 points) Does $w$ lie in the subspace spanned by $x^{(1)}, \ldots, x^{(n)}$?

(2) (3 points) Does $w$ lie in the cone spanned by $x^{(1)}, \ldots, x^{(n)}$?

(3) (3 points) Does $w$ lie in the cone spanned by $y^{(1)} x^{(1)}, \ldots, y^{(n)} x^{(n)}$?