\[
\begin{align*}
\min & \quad z_1 + z_2 \\
\text{subject to:} & \quad x_1^2 + x_2^2 - 2 = 0 \\
& \quad x_1 \leq 0 \\
\end{align*}
\]

Optimal solution: \((-1, -1)\) (from any other point we can decrease the solution value while staying on the circle).

KKT Conditions:

Lagrangian \( \mathcal{L}(x, \lambda) = z_1 + z_2 - \lambda_1 z_1 + \nu_1 (x_1^2 + x_2^2 - 2) \)

1. \( \nabla_x \mathcal{L}(x, \lambda) = 0 \):
   \[
   1 - \lambda_1 + 2\nu_1 x_1 = 0 \\
   1 + 2\nu_1 x_2 = 0.
   \]

2. Feasibility: \( x_1 \leq 0, \quad x_1^2 + x_2^2 = 2 \)

3. Complementarity slackness: \( \lambda_1 z_1 = 0 \)
   \[
   \nu_1 (x_1^2 + x_2^2 - 2) = 0.
   \]

Since at \( x^* = (-1, -1) \), \( z_1 \leq 0 \) is NOT active, \( \lambda_1 = 0 \).

Also, \( \nu_1 = -\frac{1}{2x_1^*} = \frac{1}{2} \)