Probability review I:
Probability spaces, events, and conditioning

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Probability spaces

You roll two dice.
What is the probability they add to 10?
Probability spaces

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What is the probability they add to 10?

The probability space has two components:

1. Sample space (space of outcomes).

2. Probabilities of outcomes, summing to 1.
Events

Probability space:
- Outcomes: $\Omega = \{\text{all possible pairs of dice rolls}\}$
- Every pair $z = (z_1, z_2) \in \Omega$ has probability $1/36$. 
Events

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Event of interest: the two dice add up to 10.
Another example

A drawer has 3 blue socks and 3 red socks. You pull out two socks at random. What is the probability they match?
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Outcome space $\Omega = \{(b, b), (b, r), (r, b), (r, r)\}$.

Event of interest: $A = \{(b, b), (r, r)\}$. 
Once again, roll two dice.

- Outcome space $\Omega = \{1, 2, 3, 4, 5, 6\}^2$
- All outcomes equally likely: probability $1/36$
Multiple events

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- Outcome space $\Omega = \{1, 2, 3, 4, 5, 6\}^2$
- All outcomes equally likely: probability $1/36$

Two events of interest:

- $A =$ first roll is a four
- $B =$ the sum is ten

Event that they both occur: $A \cap B$
Conditioning

Roll two dice.

• Event $A$: first roll is a four
• Event $B$: sum is ten
Conditioning

Roll two dice.

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- Event $B$: sum is ten

Conditional probability

$$\Pr(B|A) = \text{probability that } B \text{ occurs, given that } A \text{ occurs}$$
The conditioning formula

Roll two dice.

- Event $A$: first roll is a four
- Event $B$: sum is ten

Formula: $\Pr(A \cap B) = \Pr(A) \Pr(B|A)$
Bayes’ rule

Roll two dice.

- Event $A$: first roll is a four. $\Pr(A) = 1/6$.
- Event $B$: sum is ten. $\Pr(B) = 1/12$.
- Conditional probability $\Pr(B|A) = 1/6$. 
Bayes’ rule

Roll two dice.

- Event $A$: first roll is a four. $\Pr(A) = \frac{1}{6}$.
- Event $B$: sum is ten. $\Pr(B) = \frac{1}{12}$.
- Conditional probability $\Pr(B|A) = \frac{1}{6}$.

If we find out $B$ occurred, how does it alter the probability of $A$?
Bayes’ rule

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Bayes’ rule: $\Pr(A|B) = \Pr(A) \times \frac{\Pr(B|A)}{\Pr(B)}$