CSE 20: Problem Set #2

1. (a) Which of the following is the negation of the statement “For all odd primes \(p, q\) with \(p < q\), there exist positive non-primes \(r, s\) with \(r < s\), such that \(p^2 + q^2 = r^2 + s^2\)?”

   (0) For all odd primes \(p, q\) with \(p < q\), there exist positive non-primes \(r, s\) with \(r < s\), such that \(p^2 + q^2 \neq r^2 + s^2\).
   (1) There exist odd primes \(p, q\) with \(p < q\), such that for all positive non-primes \(r, s\) with \(r < s\), we have \(p^2 + q^2 = r^2 + s^2\).
   (2) There exist odd primes \(p, q\) with \(p < q\), such that for all positive non-primes \(r, s\) with \(r < s\), we have \(p^2 + q^2 \neq r^2 + s^2\).
   (3) For all odd primes \(p, q\) with \(p < q\) and for all positive non-primes \(r, s\) with \(r < s\), it is true that \(p^2 + q^2 \neq r^2 + s^2\).
   (4) There exist odd primes \(p, q\) with \(p < q\) and there exist positive non-primes \(r, s\) with \(r < s\) such that \(p^2 + q^2 \neq r^2 + s^2\).

(b) Which of the following is an unsolved conjecture?

   (0) \(\exists n \in \mathbb{N}, 2^n + 1 \notin \mathbb{P}\)
   (1) \((\exists x, y, z, n \in \mathbb{N}^+, x^n + y^n = z^n) \iff (n = 1, 2)\)
   (2) \(\forall m \in \mathbb{N}, \exists (n \geq m, n \text{ even}, \exists p, q \in \mathbb{P}, n = p + q)\)
   (3) \(\forall m \in \mathbb{N}, \exists n \geq m, n \in \mathbb{P} \text{ and } (n+2) \in \mathbb{P}\)

   For full credit, you must not only indicate the correct answer, but also explain in detail how you arrived at this result.

2. Write down the negations of the following two statements:

   (a) \(\forall \epsilon > 0, \exists m > 0 \text{ such that } \forall n > m, |x - x_n| < \epsilon\).
   (b) \(\forall x \in \mathbb{R}, \text{ if } x(x+1) \neq 0 \text{ then } x \neq 0 \text{ and } x \neq -1\).

3. Let \(P(x)\) and \(Q(x)\) be predicates defined on the same domain \(D\). For each pair of statement forms below, explain whether or not they are logically equivalent. If not, why not? If they are equivalent, explain why.

   (a) Is \(\forall x \in D, (P(x) \land Q(x))\) equivalent to \((\forall x \in D, P(x)) \land (\forall x \in D, Q(x))\)?
   (b) Is \(\exists x \in D, (P(x) \land Q(x))\) equivalent to \((\exists x \in D, P(x)) \land (\exists x \in D, Q(x))\)?
   (c) Is \(\forall x \in D, (P(x) \lor Q(x))\) equivalent to \((\forall x \in D, P(x)) \lor (\forall x \in D, Q(x))\)?
   (d) Is \(\exists x \in D, (P(x) \lor Q(x))\) equivalent to \((\exists x \in D, P(x)) \lor (\exists x \in D, Q(x))\)?
4. Write down the negation, the converse, the inverse, and the contrapositive of the following statement, and determine which are correct.

*Any circuit that adds two n-digit binary numbers contains a half-adder and n − 1 full adders.*

It would be helpful to first express this statement formally in predicate logic.

5. The Avatars of Cassiopea have only one hand with five fingers. Thus they use the base-5 number system defined by the ordered set of digits \( D = \{ \star, \bullet, \oplus, \circ, \otimes \} \).

(a) The Great Avatar was born on Cassiopea \( \otimes \star \oplus \circ \) Earth years ago. How old is he?
(b) Your cruise ship has 87 people on board. Explain this to the Great Avatar.
(c) What is \( \bullet \circ \oplus \star \) plus \( \otimes \circ \oplus \bullet \)? What is \( \circ \star \circ \bullet \) times \( \otimes \bullet \star \bullet \)? Carry out the calculations so that the Avatars can follow — that is, without using base-10 notation — and express your answers so that they can understand them.

6. (a) Convert 750342\(_8\), given in the octal number system, to decimal (base-10).
(b) Convert FAB5\(_{16}\), given in the hexadecimal form, to octal. Show your work.
(c) Convert the numbers 145 and 77 to binary, and then add them using binary addition. Show your work.

7. Convert the hexadecimal number BADDEED into binary, octal, and decimal form. There are many rearrangements of the letters in BADDEED (e.g., BDDADEE or DADBEED). Some rearrangements produce hexadecimal numbers that are larger than BADDEED, while others produce numbers that are smaller than BADDEED. Which of the rearrangements produces the smallest number larger than BADDEED? What is its decimal value?

8. Design a circuit which adds two 3-digit binary numbers, consisting only of NAND gates.