1. The beer drinkers’ database consists of the following three relations

```
frequents | drinker | bar
serves   | bar     | beer
likes    | drinker | beer
```

The first indicates the bars each drinker frequents, the second tells what beers each bar serves, and the last indicates which beers each drinker likes to drink. Express the following queries in relational algebra.

(a) List the bars that serve a beer that Joe likes.
(b) List the drinkers who frequent at least one bar that serves a beer they like.
(c) List the drinkers who frequent only bars that serve some beer that they like.
   (Assume each drinker likes at least one beer and frequents at least one bar.)
(d) List the drinkers who frequent no bar that serves a beer that they like.

2. Consider the database consisting of one relation:

```
movie: title, director, actor.
```

Express the following queries in relational algebra.

(a) List the actors cast in no movie directed by Berto.
(b) List the actors cast only in movies by Berto.
(c) List all pairs of actors who act together in at least one movie.
(d) List the directors such that every actor is cast in one of his/her movies.
   (Assume that a movie is uniquely identified by its title.)
3. Let $R$ and $S$ be relations with attributes $AB$. Prove or disprove the following:

(a) $\pi_A(R \cup S) = \pi_A(R) \cup \pi_A(S)$
(b) $\pi_A(R \cap S) = \pi_A(R) \cap \pi_A(S)$
(c) $\pi_A(R - S) = \pi_A(R) - \pi_A(S)$

4. Consider the movie database consisting of two relations

```
movie: title, director, actor
schedule: title, theater
```

Consider the relational algebra query:

$$\pi_{\text{director}}[\sigma_{\text{theater} \neq \text{Hillcrest}}(\text{movie} \Join \text{schedule})]$$

(find the directors of all movies playing elsewhere than Hillcrest). How should a query optimizer rewrite this query to make its evaluation more efficient? Explain why the rewritten query is better.