1. Let $L$ be the language over the alphabet $\{0, 1\}$ defined by

$$L = \{w | w \text{ contains an even number of 0's and an odd number of 1's and does not contain the substring 01}\}.$$ 

Give a DFA with at most five states that recognizes $L$.

[[ Optional extra practice: (1) Is there an NFA with fewer states that also recognizes $L$? (2) Give a regular expression that describes $L$. ]]}

2. Consider the NFA $N$ over the alphabet $\{a, b, c\}$ with the state diagram shown below.

![State Diagram](image)

(a) Which of the following strings are accepted by $N$?

i. abc

ii. cbbc

iii. cbbca

iv. $\varepsilon$

(b) Write the formal definition for $N$.

[[ Optional extra practice: (1) Find a DFA that recognizes $L(N)$. (2) Write a regular expression for $L(N)$.)]]

3. Give the setup and construction steps of a proof that shows that the class of regular languages over an alphabet $\Sigma$ is closed under the operation $EvenLengthStringsOnly(L)$, defined as

$$EvenLengthStringsOnly(L) = \{w \in L \text{ such that } |w| \text{ is even}\}.$$ 

Show how your general construction works on the example language of all binary strings containing the substring 101.
4. **True or False** Briefly justify each answer.

(a) For every DFA or NFA, $M$, over $\Sigma$, $L(M) = \Sigma^*$ if and only if each state is an accept state.

(b) Whenever $R_1$ is a regular expression over the alphabet $\{a, b, c\}$, $L((R_1 \circ \emptyset) \circ c) = L((R_1 \circ \varepsilon) \circ c)$.

(c) In a proof that a language is not regular using the Pumping Lemma, we should never choose $i = 1$. (Using the standard variables from the textbooks and class where $s$ is the string, $s = xyz$, and $i$ is the number of times to repeat $y$.)

(d) For all sets $A$, $B$, if $A$ and $B$ are both nonregular then $A \cap B$ is also nonregular.

(e) For all sets $L$, $L$ is regular if and only if $L^*$ is regular.