CSE 105
THEORY OF COMPUTATION

"Winter" 2018

http://cseweb.ucsd.edu/classes/wi18/cse105-ab/
Today's learning goals

- Apply the Pumping Lemma in proofs of nonregularity
- Identify some nonregular sets

Reminder:
- Register your iClicker on Google Form for this class

Individual HW 3 due tomorrow
Group HW 3 due Saturday
Exam 1 next week (Wednesday)
Pumping Lemma

If $A$ is a regular language, then there is a number $p$ (the pumping length) where, if $s$ is any string in $A$ of length at least $p$, then $s$ may be divided into three pieces, $s = xyz$, such that

- $|y| > 0$, and
- for each $i \geq 0$, $xy^iz \in A$,
- $|xy| \leq p$. 

Sipser p. 78 Theorem 1.70
Claim: The set \( L = \{0^n1^n | n \geq 0\} \) is not regular.

Proof: Consider an arbitrary positive integer. WTS \( p \) is not a pumping length for \( L \). Consider the string \( s = 0^p1^p \). Then, \( s \) is in \( L \) and \( |s| = 2p \geq p \). Consider any division of \( s \) into three parts \( s = xyz \) with \( |y| > 0 \), \( |xy| \leq p \).

Since \( |xy| \leq p \), \( x = 0^k \), \( y = 0^m \), \( z = 0^r1^p \) with \( k + m + r = p \), and since \( |y| > 0 \), \( m > 0 \). Picking \( i = 0 \): \( xy^iz = xz = 0^k0^r1^p = 0^k+r1^p \), which is not in \( L \) because \( k + r < p \). Thus, no \( p \) can be a pumping length for \( L \) and \( L \) is not regular.
Proof strategy

To prove that a language $L$ is not regular

- Consider arbitrary positive integer $p$.
- Prove that $p$ isn't a pumping length for $L$.
- Conclude that $L$ does not have any pumping length and is therefore not regular.
Another example

Claim: The set \( \{a^n b^m a^n \mid m,n \geq 0\} \) is not regular.

Proof: Consider an arbitrary positive integer. WTS \( p \) is not a pumping length for \( L \).

Consider the string

\[ s = \text{???} \]

1. \(|s| \geq p \)?
2. \( s \) is in \( L \)?
3. No matter how we cut \( s \) into three (viable) pieces, some related string obtained by "repeating" the middle part falls out of \( L \)?
Aside…

To complete proofs with Pumping Lemma, we will need to build (useful) examples of strings with length $\geq p$ that are in a given language.

- $L_1 = \{ a^m b^n a^n \mid m, n \geq 0 \}$
- $L_2 = \{ ww \mid w \text{ is a string over } \{0,1\} \}$
- $L_3 = \{ ww^R \mid w \text{ is a string over } \{0,1\} \}$
\[ \{ a^n b^m a^n \mid m,n > 0 \} \quad 2n + m = p \]

\[ p \text{ Pos int} \]

Consider \( S = a^p b a^p \) \((n = p, m = 1)\)

Claim: \( S \) is not pumpable

Consider \( S = x y z \) with \( |x y| \leq p, |y| > 0 \)

By def of \( S \), there are nonneg ints \( k, j, t \) \((k + j + t \geq p, j > 0)\)

\[ x = a^k, y = a^j, z = a^t b a^p \]

Consider \( i = 5 \):

\[ x y^5 z = a^5 a^j a^t b a^p \text{ where } k + 5j + t > p \]
Another example

Claim: The set \( \{a^m b^m a^n \mid m, n \geq 0\} \) is not regular.

Proof: … You must pick \( s \) carefully: we want \( |s| \geq p \) and \( s \) in \( L \) and \( s \) "can't be pumped"

Which other choices of \( s \) can be used to complete the proof?

A. \( s = a^p b^p \)  
B. \( s = aba \)  
C. \( s = a^p b^p a^p \)  
D. \( s = b^p \)  
E. None of the above
And another

Claim: The set \( \{w w^R \mid w \text{ is a string over } \{0,1\} \} \) is not regular.

Proof: … You must pick \( s \) carefully: we want \(|s| \geq p\) and \( s \) in \( L \) and \( s \) "can't be pumped" … Consider \( i=\ldots \)

Which \( s \) and \( i \) let us complete the proof?

A. \( s = 0^p0^p, i=2 \)  B. \( s = 0110, i=0 \)  C. \( s = 0^p110^p, i=1 \)  D. \( s = 1^p001^p, i=3 \)

E. I don't know
How do we choose \( i \)?

**Claim:** The set \( \{0^j1^k \mid j,k \geq 0 \text{ and } j \geq k \} \) is not regular.

**Proof:** ... You must pick \( s \) carefully: we want \(|s| \geq p\) and \( s \) in \( L \) and \( s \) "can't be pumped" ... **Consider** \( i=\ldots \)

Which \( s \) and \( i \) let us complete the proof?

A. \( s = 0^p1^p, \ i=2 \)  
B. \( s = 0^p1^p, \ i=p \)  
C. \( s = 0^p1^p, \ i=1 \)  
D. \( s = 0^p1^p, \ i=0 \)  
E. I don't know
Do we always need Pumping Lemma?

Claim: The set \( L = \{ w | w \text{ has different } \#s \text{ of 0s and 1s OR has a 1 before a 0} \} \) is nonregular.

Proof:

Since class of regular languages is closed under complement...

\( \overline{L} = \{ w | \text{same } \# \text{ of 0s and 1s AND no 1s before 0s} \} = \{ 0^n \ 1^n | n \geq 0 \} \)
\[ L = \{ \emptyset \} \cup \{ \{0,1\} \} \text{ being nonregular guarantees } L \text{ is nonregular} \]
Regular sets: not the end of the story

- Many **nice / simple / important** sets are not regular
- Limitation of the finite-state automaton model
  - Can't "count"
  - Can only remember finitely far into the past
  - Can't backtrack
  - Must make decisions in "real-time"
- We know computers are more powerful than this model…

Which conditions should we relax?
For next time

• Work on Individual HW3  due Tuesday

Pre class-reading for Wednesday: pages 111-112.