CSE 105
THEORY OF COMPUTATION

"Winter" 2018

http://cseweb.ucsd.edu/classes/wi18/cse105-ab/
Today's learning goals

- Design NFA recognizing a given language
- Convert an NFA (with or without spontaneous moves) to a DFA recognizing the same language
Formal definition?

NFA

\[
L(M_1) = \{ w \in \{0,1\}^* \mid w = 0^n \text{ or } 1^n \text{ for } n \geq 0 \} 
\]

ends with 00

ends with 11

end with 11


L(M_2) = \{ w \in \{0,1\}^* \mid \text{last two chars of } w \text{ are equal} \}
Simulating NFA with DFA

Not quite a closure proof, but …

**Proof:**

**Given** name variables for sets, machines assumed to exist.

**WTS** state goal and outline plan.

**Construction** using objects previously defined + new tools working towards goal. Give formal definition and explain.

**Correctness** prove that construction works.

**Conclusion** recap what you've proved.
Simulating NFA with DFA

For any language recognized by an NFA, there is a DFA that recognizes this language.

**Proof:**

**Given** A, a language recognized by N = (Q,Σ,δ,q0,F) a NFA

**WTS** there is some DFA M with L(M) = A

**Construction**

**Correctness**

**Conclusion**
Idea of construction

Track set of possible states NFA might be in.
Which states can this NFA be in before first input symbol is read?

A. q0
B. any state
C. q0, q1
D. q0, q4
E. q0, q1, q4
Subset construction

**Given** A, a language recognized by $N = (Q, \Sigma, \delta, q_0, F)$ a NFA

**WTS** there is some DFA $M$ with $L(M) = A$

**Construction** Define $M = (Q', \Sigma, \delta', q_0', F')$ with

- $Q' = \text{the power set of } Q = \{ X \mid X \text{ is a subset of } Q \}$
- $q_0' = \{ \text{states } N \text{ can be in before first input symbol read} \}$
- $F' = \{ \}$
- $\delta' (\quad) =$
Subset construction

Given A, a language recognized by N = (Q, Σ, δ, q₀, F) a NFA

WTS there is some DFA M with L(M) = A

Construction Define M = (Q', Σ, δ', q₀', F') with

• Q' = the power set of Q = { X | X is a subset of Q }  
  ex: {q₀, q₁, q₄}
• q₀' = { q₀ } U δ((q₀, ε)) …
• F' = { … }
• δ' ( … ) =
Subset construction

Given $A$, a language recognized by $N = (Q, \Sigma, \delta, q_0, F)$ a NFA

WTS there is some DFA $M$ with $L(M) = A$

Construction Define $M = (Q', \Sigma, \delta', q'_0, F')$ with

- $Q' = \text{the power set of } Q = \{ X \mid X \text{ is a subset of } Q \}$
- $q'_0 = \{ q_0 \} \cup \delta((q_0, \epsilon))$ ...
- $F' = \{ \ldots \}$
- $\delta' ((X, x)) = \{ q \in Q \mid q \text{ is in } \delta((r, x)) \text{ for some } r \in X \text{ or is accessible via spontaneous moves} \}$

Types?
From NFA to DFA

\[ \{ q_0, q_1, q_2, q_3 \} \]

\[ \{ q_0, q_1, q_2, q_3 \} \]

Ex.
Subset construction

Given A, a language recognized by $N = (Q, \Sigma, \delta, q_0, F)$ a NFA, WTS there is some DFA $M$ with $L(M) = A$

Construction Define $M = (Q', \Sigma, \delta', q_0', F')$ with
- $Q' = \text{the power set of } Q = \{ X \mid X \text{ is a subset of } Q \}$
- $q_0' = \{ q_0 \} \cup \delta((q_0, \varepsilon)) \ldots$
- $F' = \{ \text{guarantee at least one computation is successful} \}$
- $\delta' ((X, x)) = \{ q \in Q \mid q \text{ is in } \delta((r, x)) \text{ for some } r \in X \text{ or is accessible via spontaneous moves} \}$
What does it mean for a set of states $X$ to guarantee at least one computation is successful?

A. $X$ is a subset of $F$
B. $X = F$
C. $X \cap F$ is nonempty
D. $X$ is an element of $F$
E. None of the above.

Example: is $\{q_0, q_1, q_4\}$ in $F$?

\[ \text{No} \]

i.e. $\{X \in \mathcal{P}(Q) \mid X \cap F \neq \emptyset\}$
Subset construction examples
Theorem: For each language $L$,

$L$ is recognizable by some DFA

iff

$L$ is recognizable by some NFA
DFA equiv NFA equiv RegExp

**Theorem:** For each language $L$,

$L$ is recognizable by some DFA
iff
$L$ is recognizable by some NFA
iff
$L$ is describable by some regular expression
For next time

• Work on Individual HW2  due Tuesday

Pre class-reading for Wednesday: Example 1.56 on page 68