Today's learning goals

- Trace the computation of a Turing machine on given input
- Describe the language recognized by a Turing machine
- Determine if a Turing machine is a decider
- Give an implementation-level description of a Turing machine
Turing machine computation

- Read/write head starts at leftmost position on tape
- Input string written on leftmost squares of tape, rest is blank
- Computation proceeds according to transition function:
  - Given current state of machine, and current symbol being read
  - the machine
    - transitions to new state
    - writes a symbol to its current position (overwriting existing symbol)
    - moves the tape head L or R
- Computation ends if and when machine enters either the accept or the reject state.
Language of a Turing machine

$L(M) = \{ w \mid \text{computation of } M \text{ on } w \text{ halts after entering the accept state} \}$

i.e. $L(M) = \{ w \mid w \text{ is accepted by } M \}$
Language of a TM

\[ L(M) = \{ w \mid M \text{ accepts } w \} \]

Which of the following is not always true?

A. If \( w \) is in \( L(M) \) then the computation of \( M \) on \( w \) halts and accepts.

B. If the computation of \( M \) on \( w \) halts and rejects, then \( w \) is not in \( L(M) \).

C. If \( w \) is not in \( L(M) \) then the computation of \( M \) on \( w \) halts and rejects.
Computation

To trace DFAs: enough to list states.
To trace NFAs: tree of possible current states (incl. spontaneous moves)
To trace PDAs: tree of possible computations incl. state + stack

- Current state
- Current tape contents up to (finite) point after which all blank
- Current location of read/write head

\[ u \ q \ v \]

current state is q

current tape contents are \( uv \) (and then all blanks)

current head location is first symbol of \( v \)
Special configurations

For input string $w$

- Starting configuration $q_0 w$
- Accepting configuration $u q_{acc} v$
- Rejecting configuration $u q_{rej} v$

current state is $q$
current tape contents are $uv$ (and then all blanks)
current head location is first symbol of $v$
Language of a TM

$L(M) = \{ w \mid M \text{ accepts } w \}$

$= \{ w \mid \text{there is a sequence of configurations of } M$

where $C_1$ is start configuration of $M$ on input $w$, each $C_i$ yields $C_{i+1}$ and $C_k$ is accepting configuration}$

"The language of $M$"

"The language recognized by $M"
An example

$L = \{ w#w \mid w \text{ is in } \{0,1\}^* \}$

*We already know that $L$ is*

• not regular
• not context-free

*We will prove that $L$ is*

the language of some Turing machine
Implementation-level description

\[ L = \{ w#w \mid w \text{ is in } \{0,1\}^* \} \]

Idea for Turing machine

- Zig-zag across tape to corresponding positions on either side of '#' to check whether these positions agree. If they do not, or if there is no '#', reject. If they do, cross them off.
- Once all symbols to the left of the '#' are crossed off, check for any un-crossed-off symbols to the right of '#': if there are any, reject; if there aren't, accept.
**Implementation-level description**

Zig-zag across tape to corresponding positions on either side of '#' to check whether these positions agree. If they do not, or if there is no '#', *reject*. If they do, cross them off.

Once all symbols to the left of the '#' are crossed off, check for any un-crossed-off symbols to the right of '#': if there are any, *reject*; if there aren't, *accept*.

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**State diagram**

- \( q_1 \)
- \( 0 \rightarrow ?, ? \)
- \( 1 \rightarrow ?, ? \)
- \( \# \rightarrow ?, ? \)
- \( \_ \rightarrow ?, ? \)
Formal definition

\[Q = \{q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_{\text{accept}}, q_{\text{reject}}\}\]

\[\Sigma = \{0, 1, \#\}\]

\[\Gamma = \{0, 1, \#, x, _\}\]

All missing transitions have output \((q_{\text{reject}}, _, R)\)

Fig 3.10 in Sipser
Configuration $u \ q \ v$

for current tape $uv$ (and then all blanks), current head location is first symbol of $v$, current state $q$
Computation on input 0#
For next time

Group HW4 due Saturday, February 17

For Friday, pre-class reading: pp. 184-185