Three-View Geometry

Computer Vision II
CSE 252B
Trifocal Plane
Trifocal Tensor

• 3x3x3 tensor
• 27 elements, 18 degrees of freedom
  – 33 degrees of freedom (3 camera projection matrices) minus 15 degrees of freedom (3D projective transformation)
• Uses tensor notation
  – Einstein summation
• Retrieve fundamental matrices and camera projection matrices
## Trilinear Relations

<table>
<thead>
<tr>
<th>Correspondence</th>
<th>Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>three points</td>
<td>$x^i x'^j x''^k \epsilon_{jqs} \epsilon_{krt} \mathcal{T}<em>{i}^{qr} = 0</em>{st}$</td>
</tr>
<tr>
<td>two points, one line</td>
<td>$x^i x'^j l''<em>r \epsilon</em>{jqs} \mathcal{T}_{i}^{qr} = 0_s$</td>
</tr>
<tr>
<td>one point, two lines</td>
<td>$x^i l'_q l''<em>r \mathcal{T}</em>{i}^{qr} = 0$</td>
</tr>
<tr>
<td>three lines</td>
<td>$l_p l'_q l''<em>r \epsilon^{piw} \mathcal{T}</em>{i}^{qr} = 0^w$</td>
</tr>
</tbody>
</table>
3 Points

• Point-Point-Point
2 Points, 1 Line

• Point-Line-Point
  – Note: image line must pass through corresponding image point
1 Point, 2 Lines

• Point-Line-Line
  – Note: image lines do not need to correspond, but must pass through corresponding image points
3 Lines

• Line-Line-Line
Trifocal Tensor, Minimal Solution

- 6 image point correspondences between three images
  - The last 4 image points must be in general position
    - 6 choose 4 = 15 combinations (rearrange, if needed)
- Carlsson-Weinshall duality (interchange camera and points)
  - Dualize first 2 image points
    - Results in 3 dual point correspondences
    - Additional 4 dual point correspondences are 2D projective basis
  - Calculate reduced dual fundamental matrix (5 degrees of freedom) from 7 dual point correspondences
    - 1 or 3 solutions
- Solve for three camera projection matrices
  - 5 3D points are 3D projective basis
  - 1 or 3 solutions for 6th 3D point using dual fundamental matrix
- Calculate trifocal tensor from three camera projection matrices
  - 1 or 3 solutions
Trifocal Tensor Estimation

- **Outlier rejection (MSAC)**
  - 6-point algorithm (previous slide)
  - Sampson error
- **Linear estimate (DLT)** \( x_i ^{q} l' q'^{r} T^{qr}_{i} = 0 \)
  - Enforce constraints on resulting trifocal tensor
    - 18 degrees of freedom
- **Nonlinear estimate (L-M)**
  - Cameras
    - Minimal parameterization
      - 3 angle-axis vectors (9 parameters) + 10-vector (with unity norm; 9 parameters)
    - Retrieve camera projection matrices from trifocal tensor
      - First camera projection matrix is canonical camera
  - 3D scene points
    - Triangulation of Sampson correct points
  - Adjust second and third camera projection matrices, and 3D scene points such that reprojection error is minimized
Mapping Under Trifocal Tensor

- Example: lines in any two images to line in the other image

\[ l_p l_q l_r e^{piw} T_i^{qr} = 0^w \]
Mapping Under Trifocal Tensor

- Example: points in any two images to point in the other image

\[ x^i x'^j x''^k e_{jqs} e_{krt} T_i^{qr} = 0_{st} \]