CSE 158 – Lecture 4

Web Mining and Recommender Systems

More Classifiers

Last lecture...

How can we predict **binary** or **categorical** variables?

$$f(\text{data}) \stackrel{?}{\to} \text{labels}$$

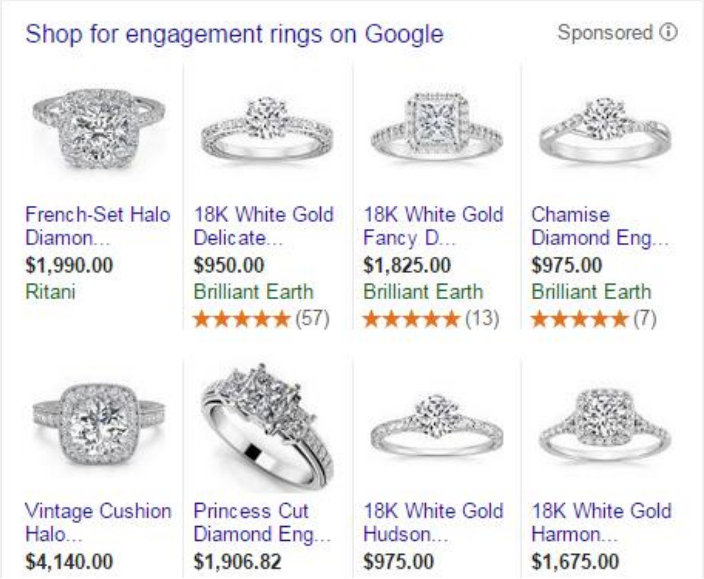
{0,1}, {True, False}

{1, ... , N}

Last lecture...

Will I **purchase** this product?

(yes)



Will I **click on** this ad?

(no)

Last lecture…

- **Naïve Bayes**
  - Probabilistic model (fits $p(label|data)$)
  - Makes a conditional independence assumption of the form $(feature_i \perp\!\!\!\perp feature_j | label)$ allowing us to define the model by computing $p(feature_i | label)$ for each feature
  - Simple to compute just by counting
- **Logistic Regression**
  - Fixes the "double counting" problem present in naïve Bayes
- **SVMs**
  - Non-probabilistic: optimizes the classification error rather than the likelihood

1) Naïve Bayes

posterior prior likelihood

$$p(label|features) = \frac{p(label)p(features|label)}{p(features)}$$

evidence

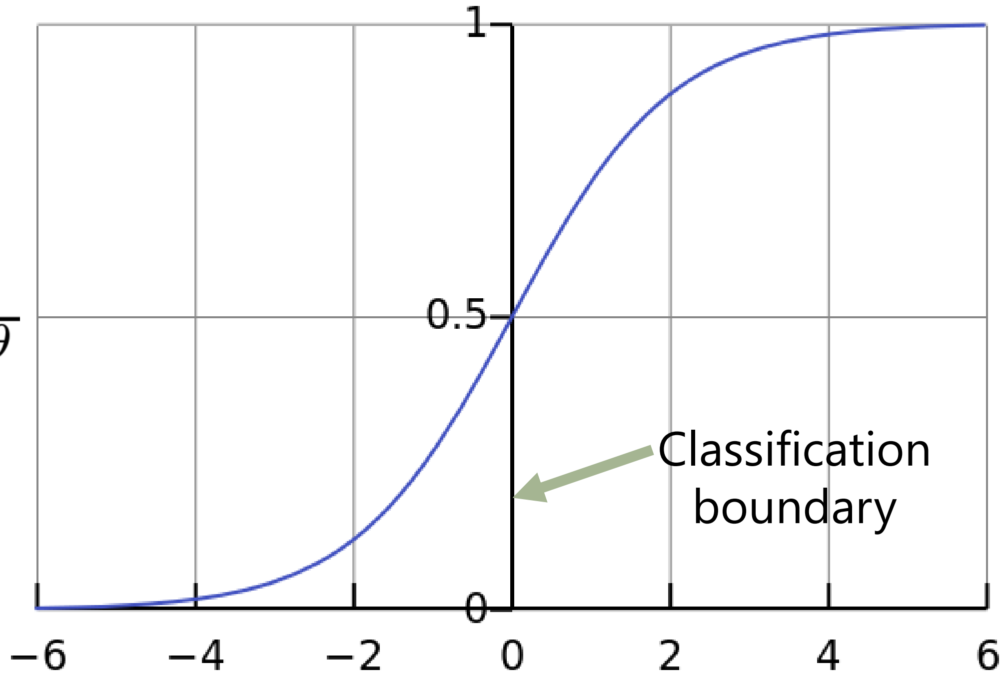
due to our conditional independence assumption:

$$p(label|features) = \frac{p(label)\prod_i p(feature_i|label)}{p(features)}$$

2) logistic regression

sigmoid function: $\sigma(t) = \frac{1}{1+e^{-t}}$

$$\frac{1}{1+e^{-X_i \cdot \theta}}$$



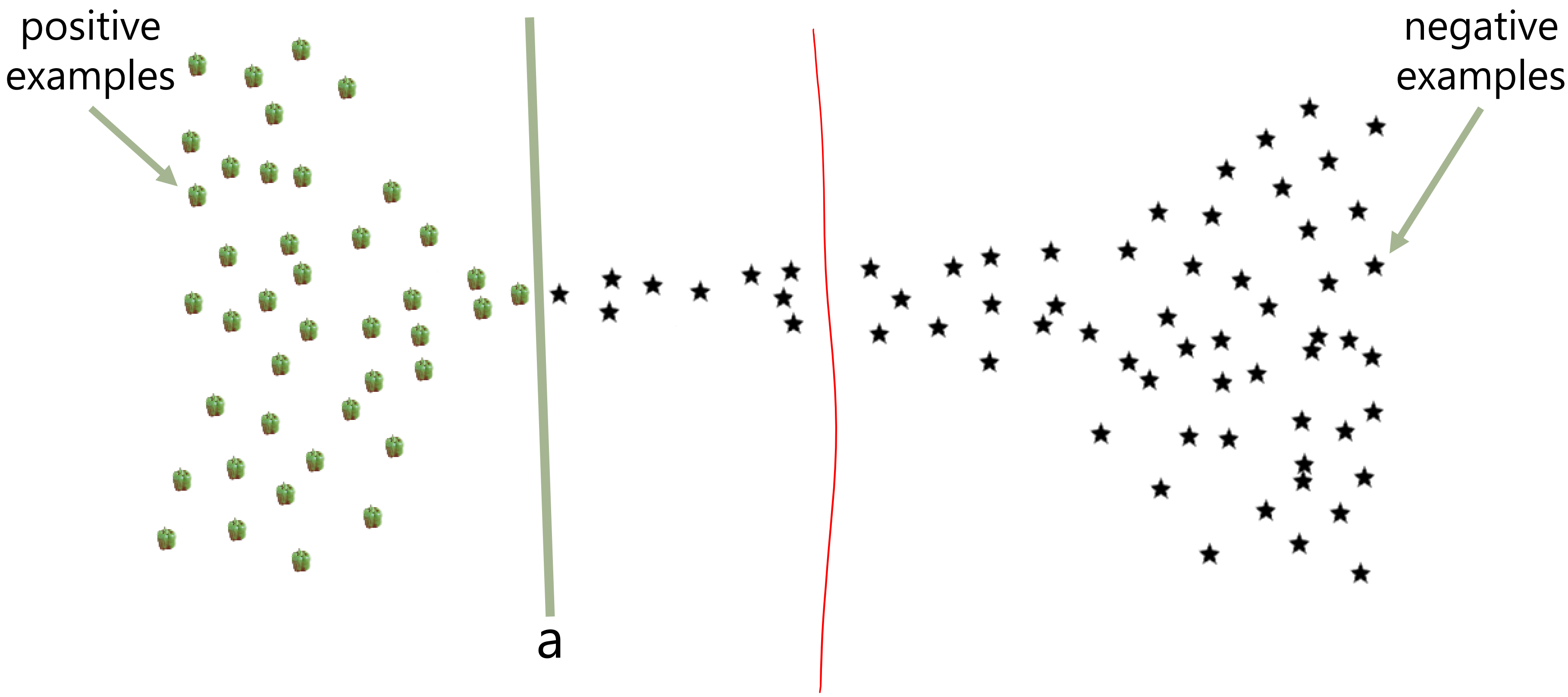
$$X_i \cdot \theta$$

Logistic regression

- Logistic regressors don’t optimize the number of “mistakes”
- No special attention is paid to the “difficult” instances – every instance influences the model
- But “easy” instances can affect the model (and in a bad way!)
- How can we develop a classifier that optimizes the number of mislabeled examples?

3) Support Vector Machines

Try to optimize the **misclassification error** rather than maximize a probability



Support Vector Machines

This is essentially the intuition behind Support Vector Machines (SVMs) – train a classifier that focuses on the "difficult" examples by minimizing the misclassification error

We still want a classifier of the form

θ_0

$$y_i = \begin{cases} 1 & \text{if } X_i \cdot \theta - \alpha > 0 \\ -1 & \text{otherwise} \end{cases}$$

But we want to minimize the number of misclassifications:

$\delta(x) = 1$ iff x is true

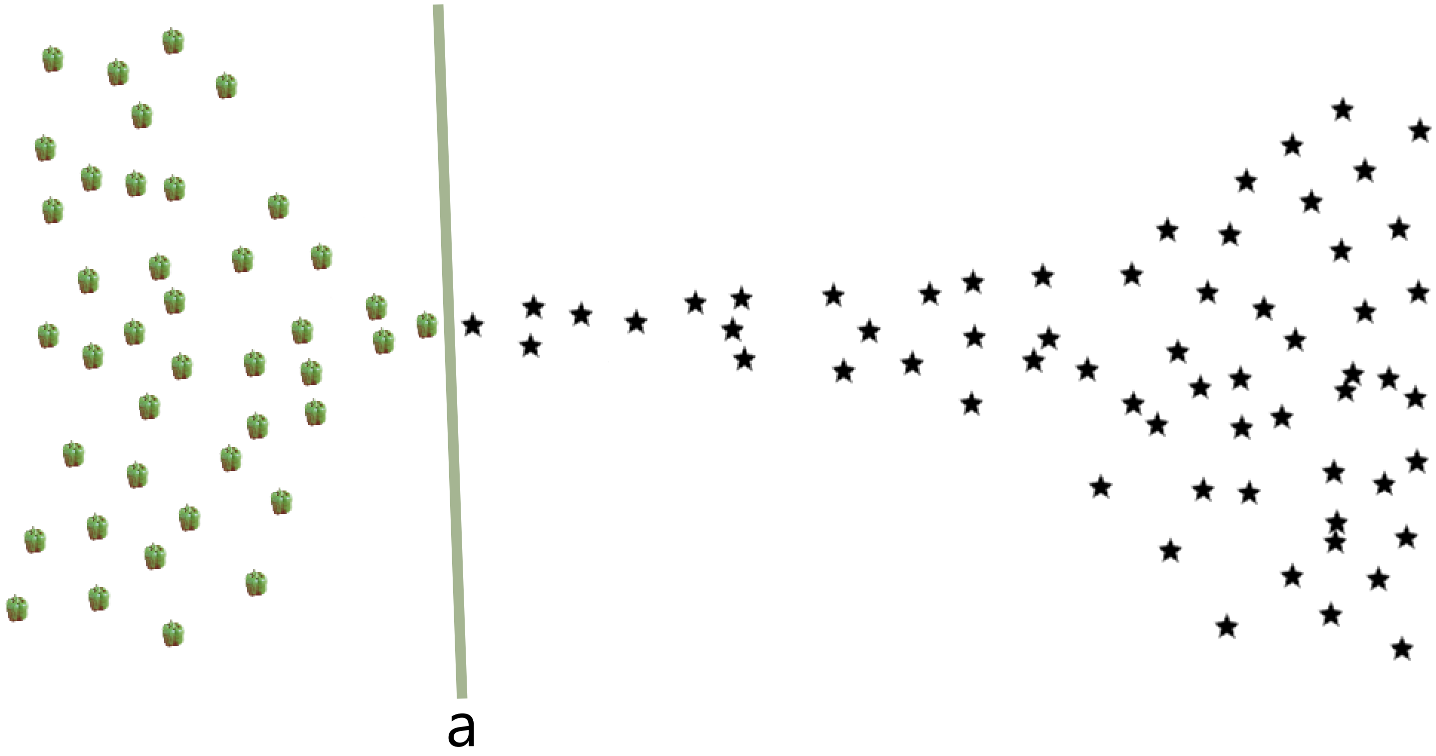
$$\arg\min_\theta \textstyle\sum_i \delta(y_i(X_i \cdot \theta - \alpha) \leq 0)$$

Support Vector Machines

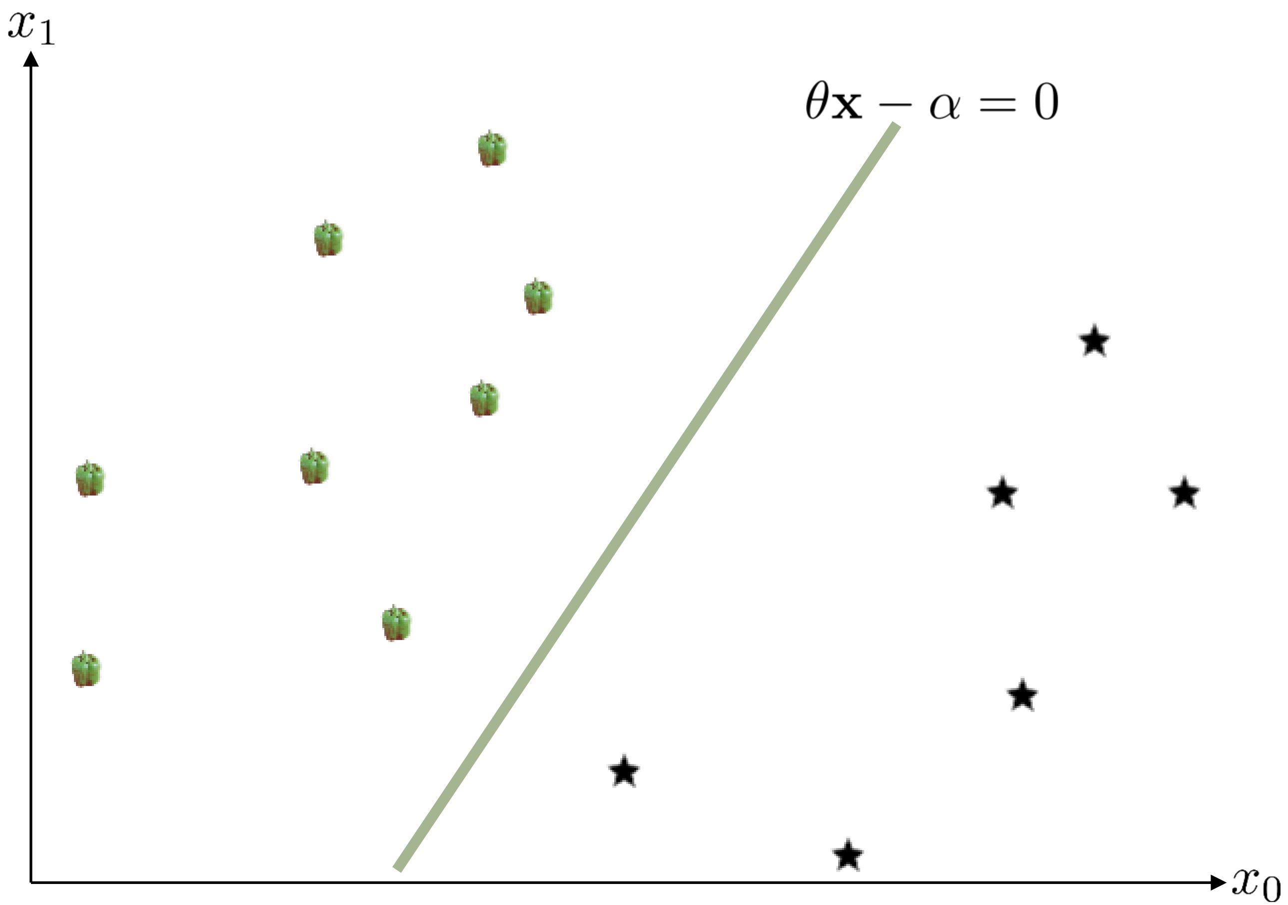
$$\arg\min_\theta \sum_i \delta(y_i(X_i \cdot \theta - \alpha) \leq 0)$$

Support Vector Machines

Simple (seperable) case: there exists a perfect classifier

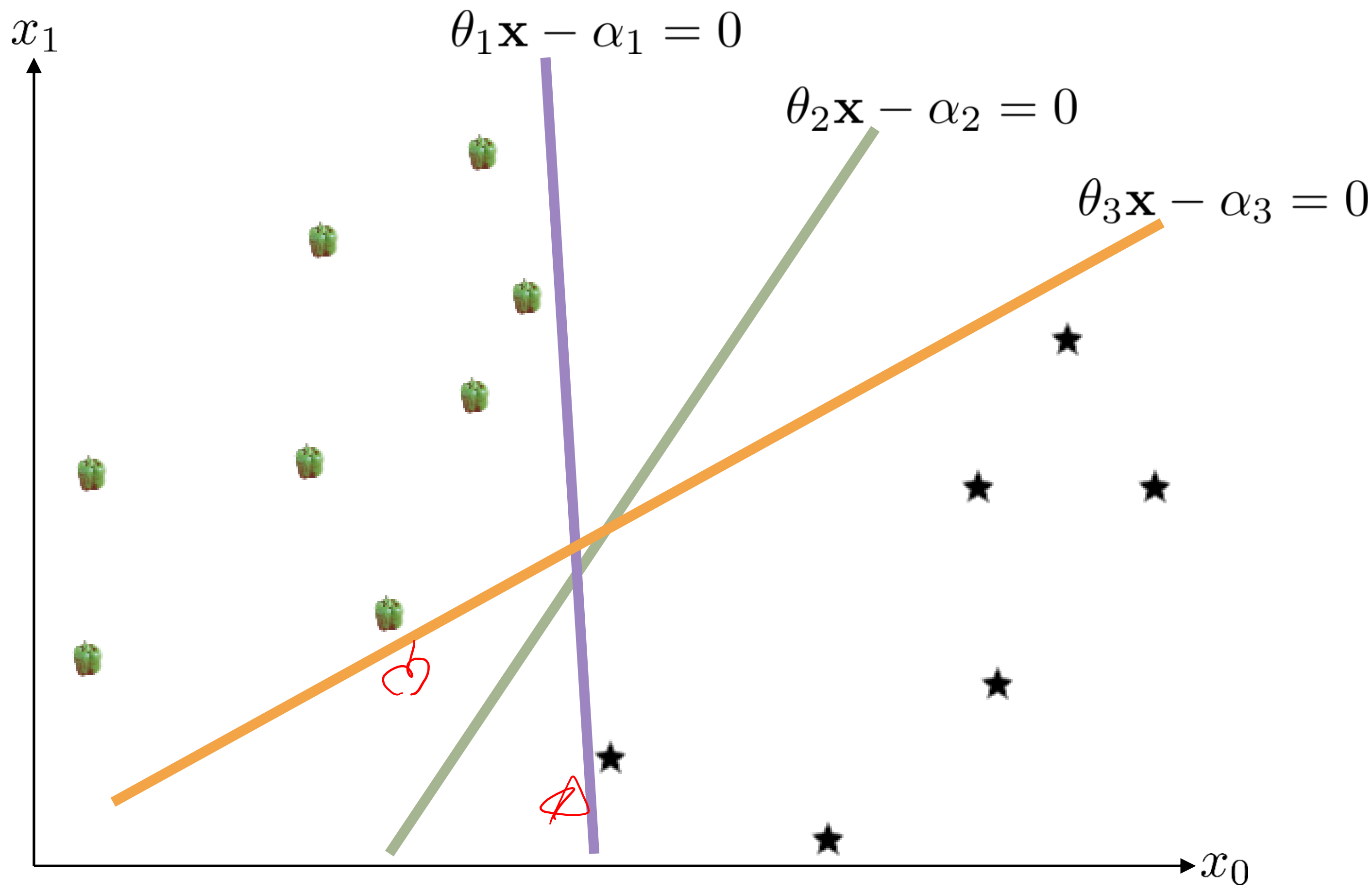


Support Vector Machines



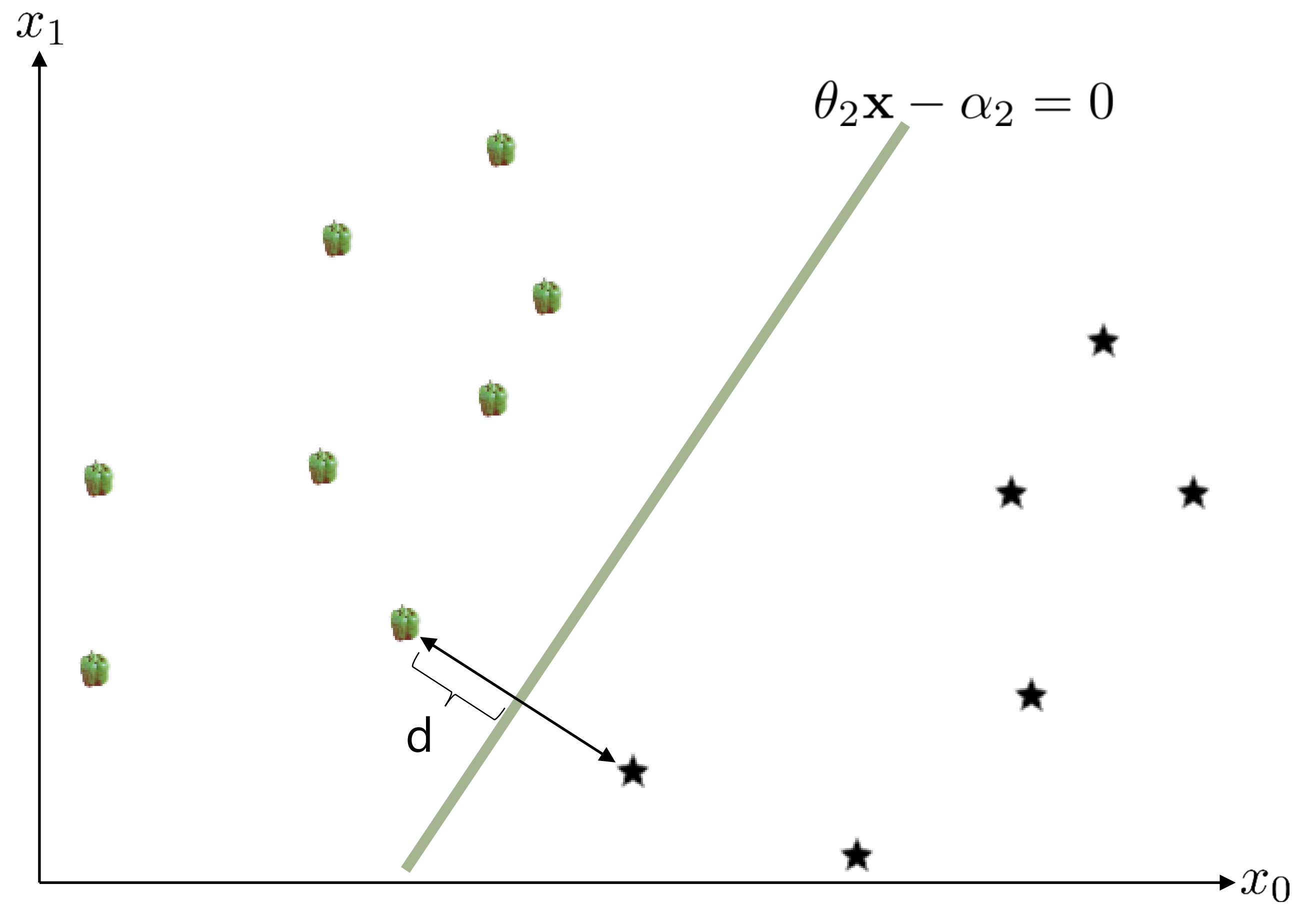
The classifier is defined by the hyperplane $\theta\mathbf{x} - \alpha = 0$

Support Vector Machines



Q: Is one of these classifiers preferable over the others?

Support Vector Machines



A: Choose the classifier that maximizes the distance to the nearest point

Support Vector Machines

Distance from a point to a line?

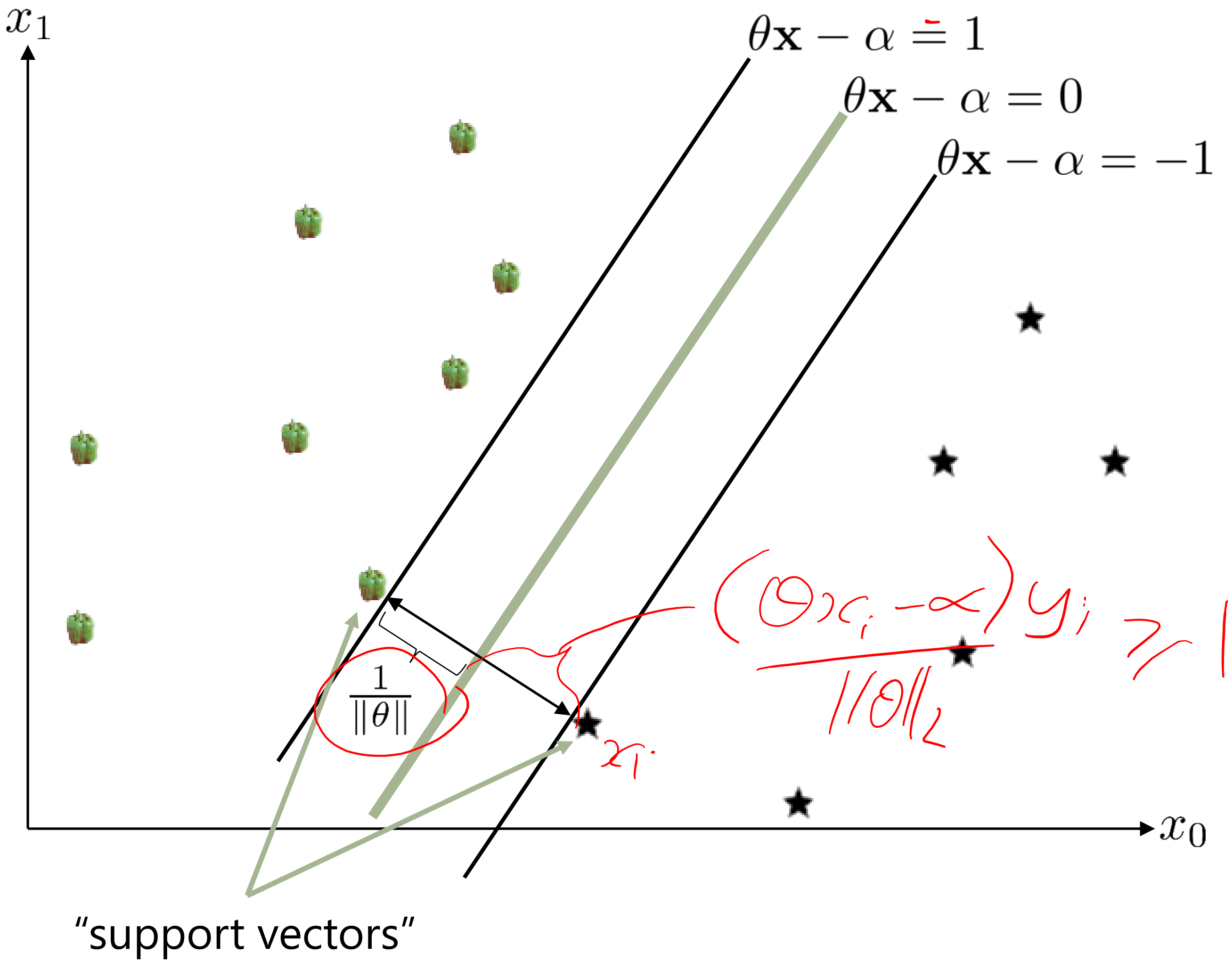
$$ax + by + c = 0 \qquad (x_0, y_0)$$

$$d(\text{ln}, \text{pt}) = \frac{|a x_0 + b y_0 + c|}{\sqrt{a^2 + b^2}}$$

$$\theta x - \alpha = 0 \qquad x_0$$

$$\frac{|\theta x_0 - \alpha|}{\sqrt{\sum_i \theta_i^2 = \|\theta\|_2}}$$

Support Vector Machines



$$\arg\min_{\theta,\alpha} \frac{1}{2}\|\theta\|_2^2$$

such that

$$\forall_i y_i(\theta \cdot X_i - \alpha) \geq 1$$

Support Vector Machines

This is known as a "quadratic program" (QP) and can be solved using "standard" techniques

$$\arg\min_{\theta,\alpha} \frac{1}{2}\|\theta\|_2^2$$

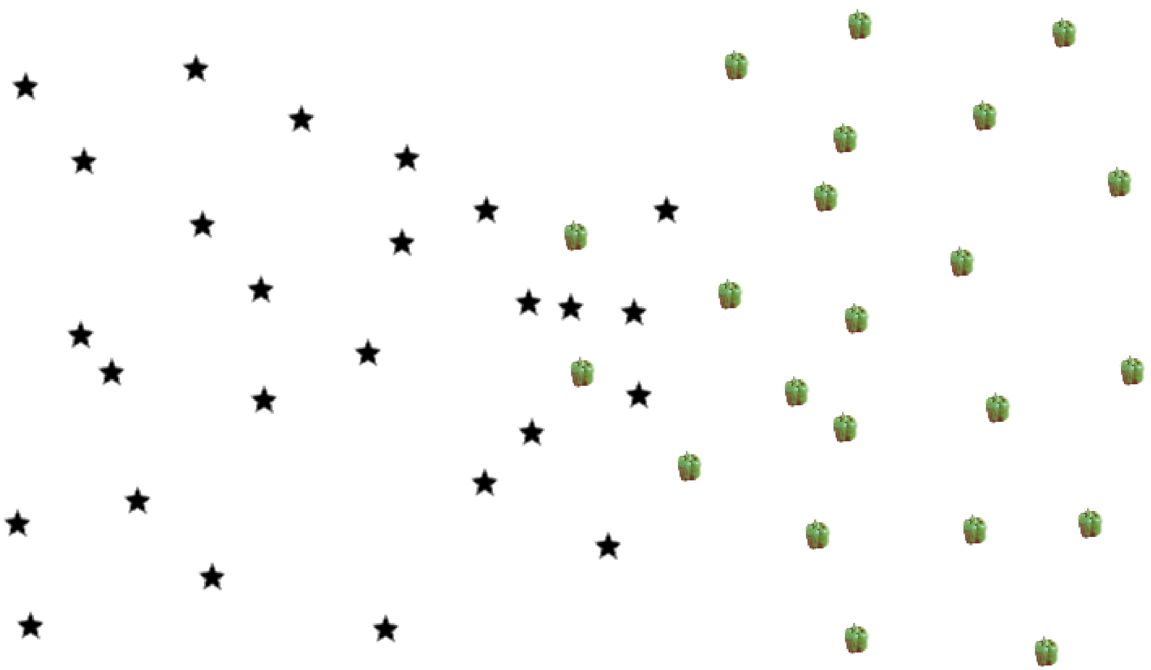
such that

$$\forall_i y_i(\theta \cdot X_i - \alpha) \geq 1$$

See e.g. Nocedal & Wright ("Numerical Optimization"), 2006

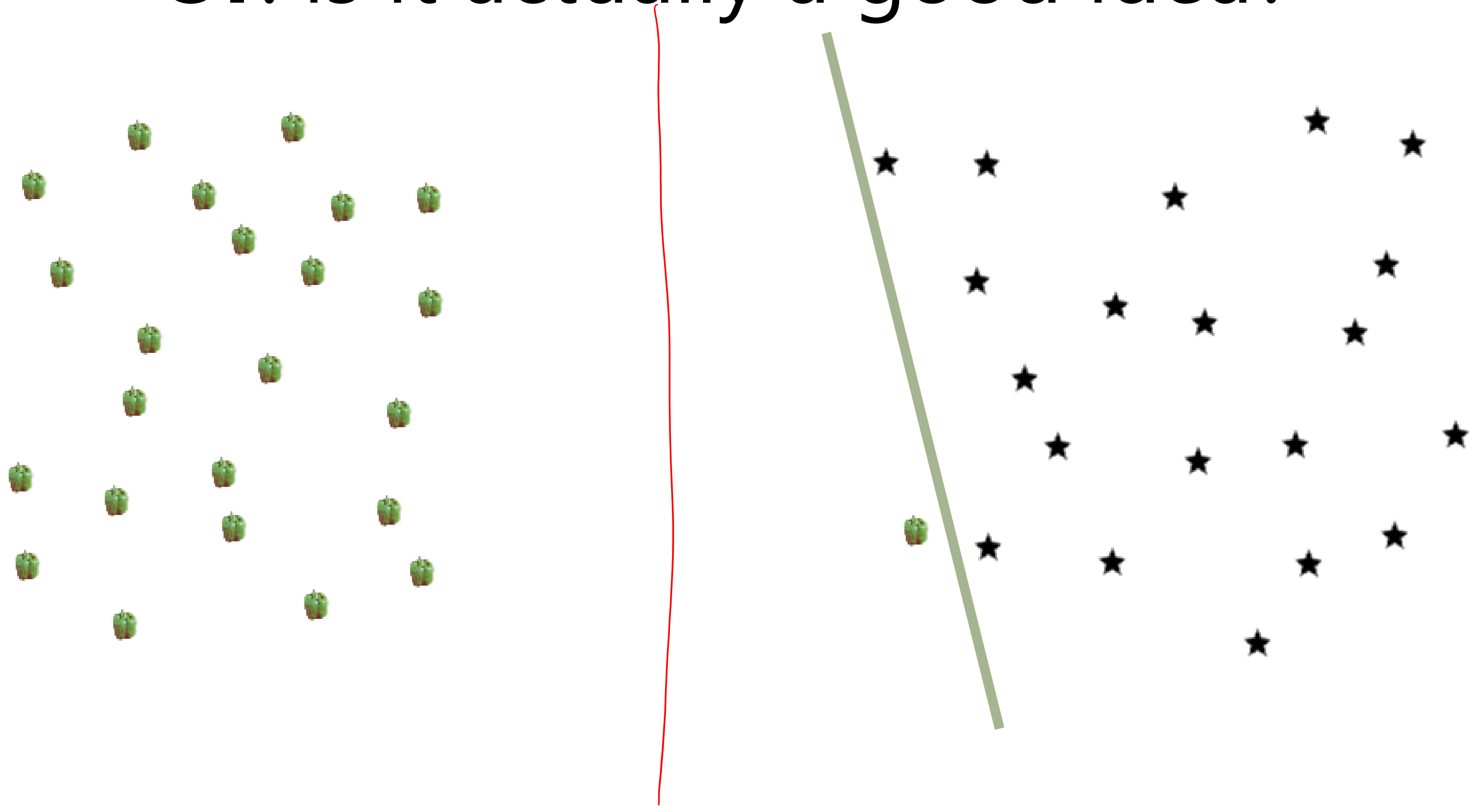
Support Vector Machines

But: is finding such a separating hyperplane even possible?



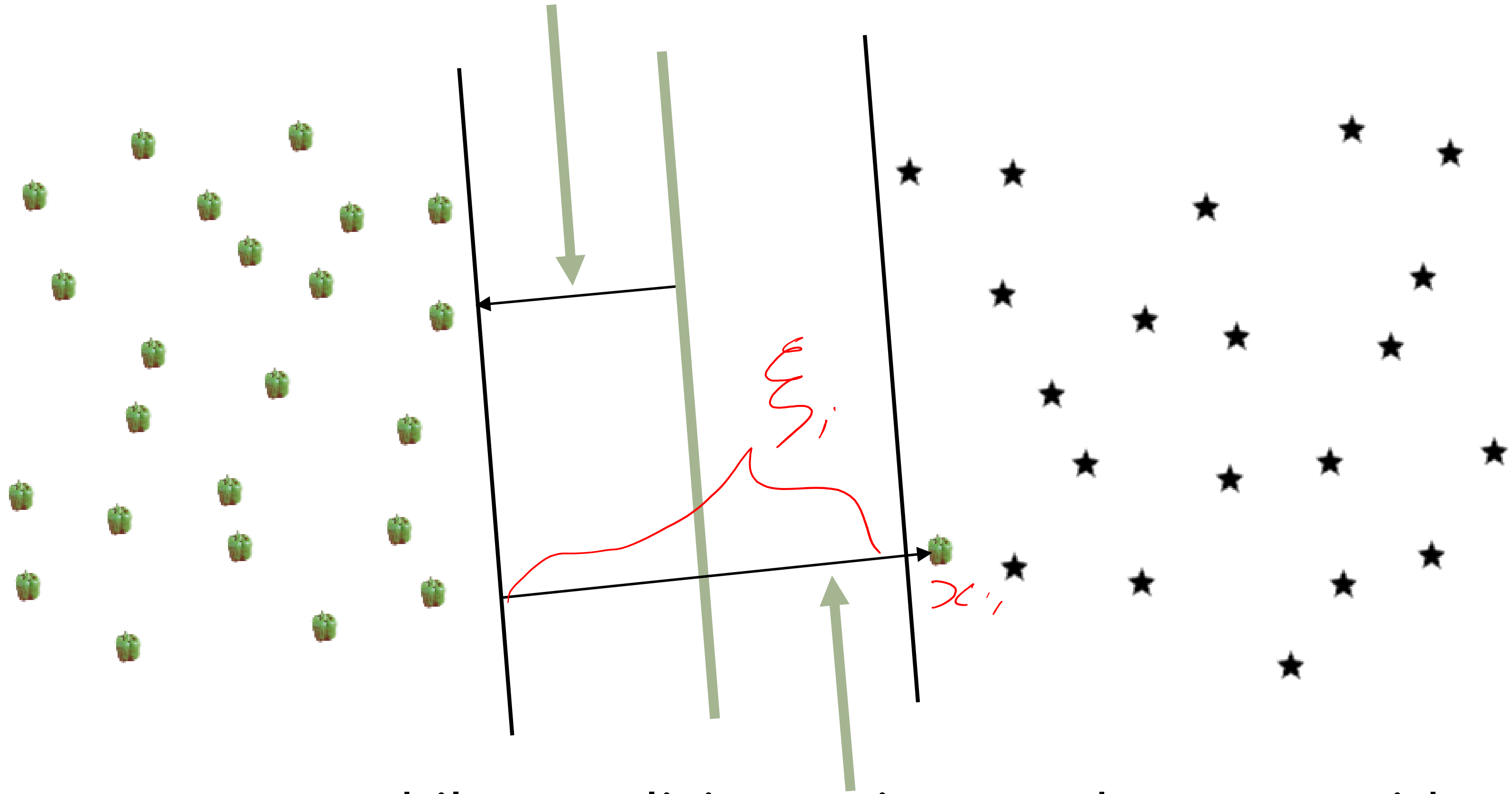
Support Vector Machines

Or: is it actually a good idea?



Support Vector Machines

Want the margin to be as wide as possible



While penalizing points on the wrong side of it

Support Vector Machines

Soft-margin formulation:

$$\arg\min_{\theta,\alpha,\ \xi > 0} \frac{1}{2}\|\theta\|_2^2 + \sum_i \xi_i$$

such that

$$\forall_i y_i(\theta \cdot X_i - \alpha) \geq 1 - \xi_i$$

Pros/cons

- **Naïve Bayes**

  ++ Easiest to implement, most efficient to "train"
  ++ If we have a process that generates feature that *are* independent given the label, it's a very sensible idea
  -- Otherwise it suffers from a "double-counting" issue

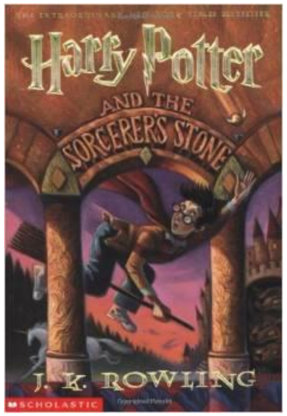
- **Logistic Regression**

  ++ Fixes the "double counting" problem present in naïve Bayes
  -- More expensive to train

- **SVMs**

  ++ Non-probabilistic: optimizes the classification error rather than the likelihood
  -- More expensive to train

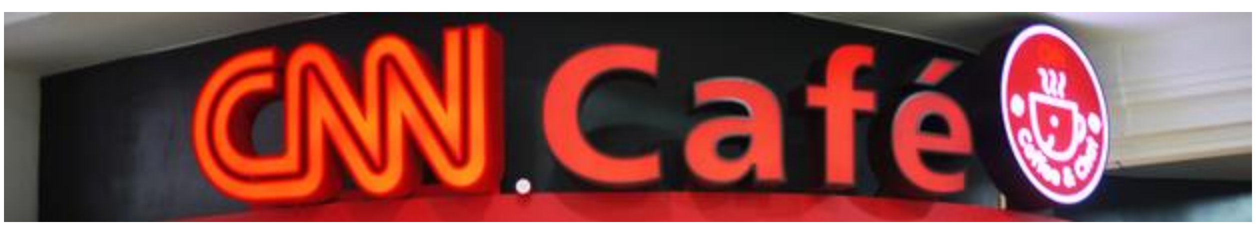
Judging a book by its cover



[0.723845, 0.153926, 0.757238, 0.983643, ...]

4096-dimensional image features

Images features are available for each book on
http://jmcauley.ucsd.edu/cse158/data/amazon/book_images_5000.json



http://caffe.berkeleyvision.org/

Judging a book by its cover

Example: train an SVM to predict whether a book is a children's book from its cover art

(code available on)
http://jmcauley.ucsd.edu/cse158/code/week2.py

Judging a book by its cover

- The number of errors we made was extremely low, yet our classifier doesn't seem to be very good – why?

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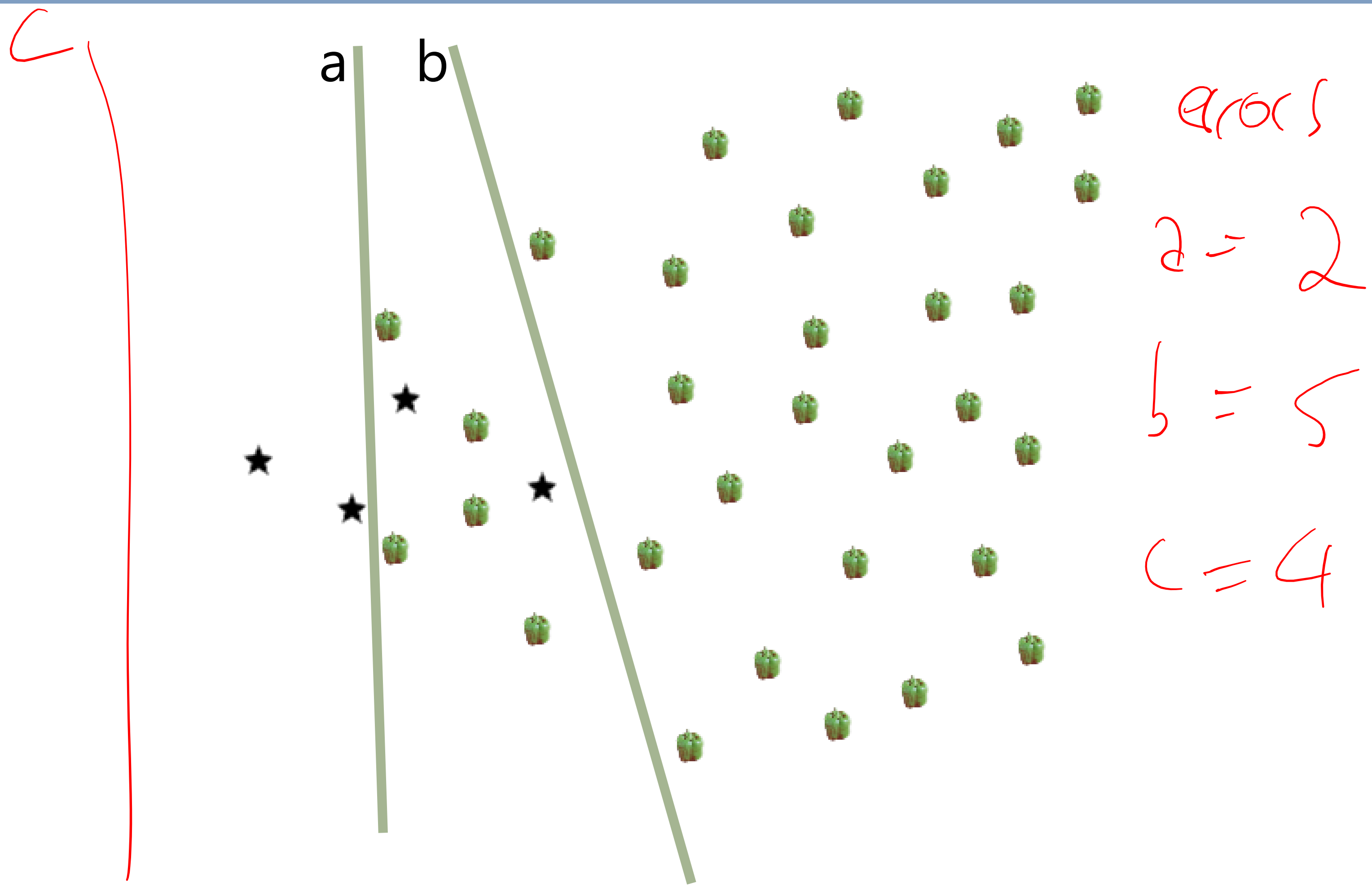
Web Mining and Recommender Systems

Evaluating Classifiers

Which of these classifiers is best?



Which of these classifiers is best?

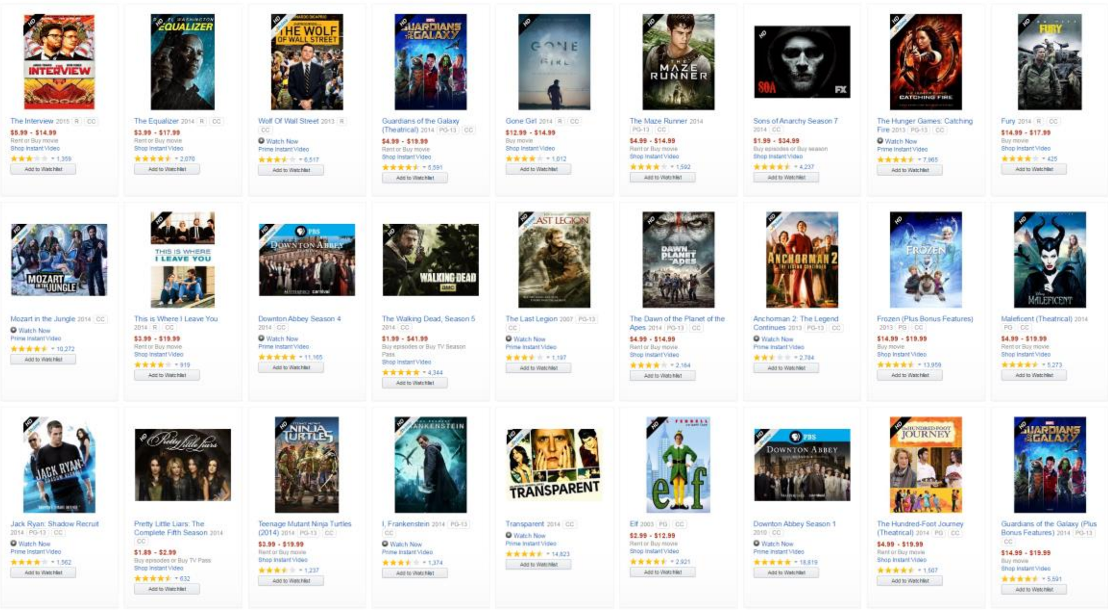
The solution which minimizes the #errors may not be the best one

Which of these classifiers is best?

1. When data are highly imbalanced

If there are far fewer positive examples than negative examples we may want to assign additional weight to negative instances (or vice versa)

e.g. will I purchase a product? If I purchase 0.00001% of products, then a classifier which just predicts "no" everywhere is 99.99999% accurate, but not very useful

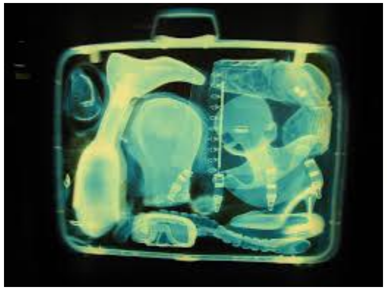
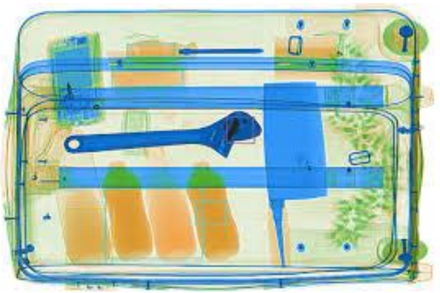


Which of these classifiers is best?

2. When mistakes are more costly in one direction

False positives are nuisances but false negatives are disastrous (or vice versa)



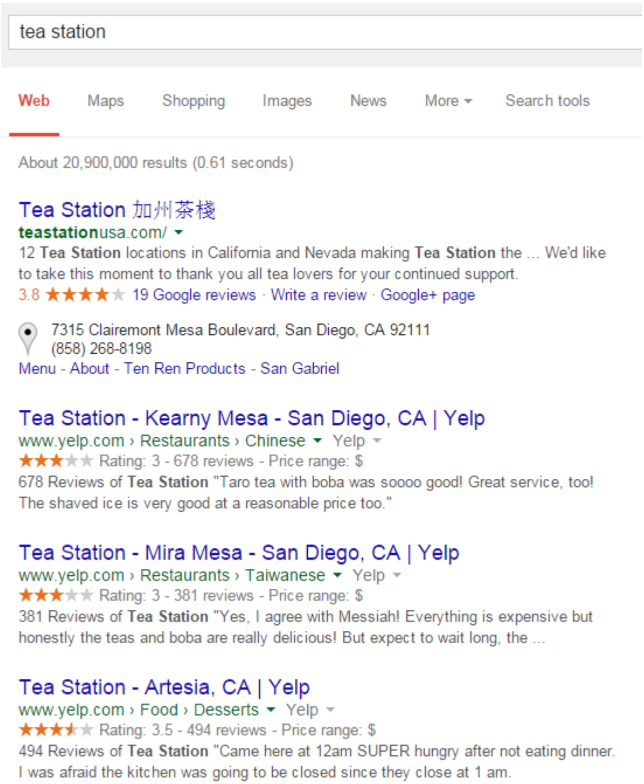



e.g. which of these bags contains a weapon?

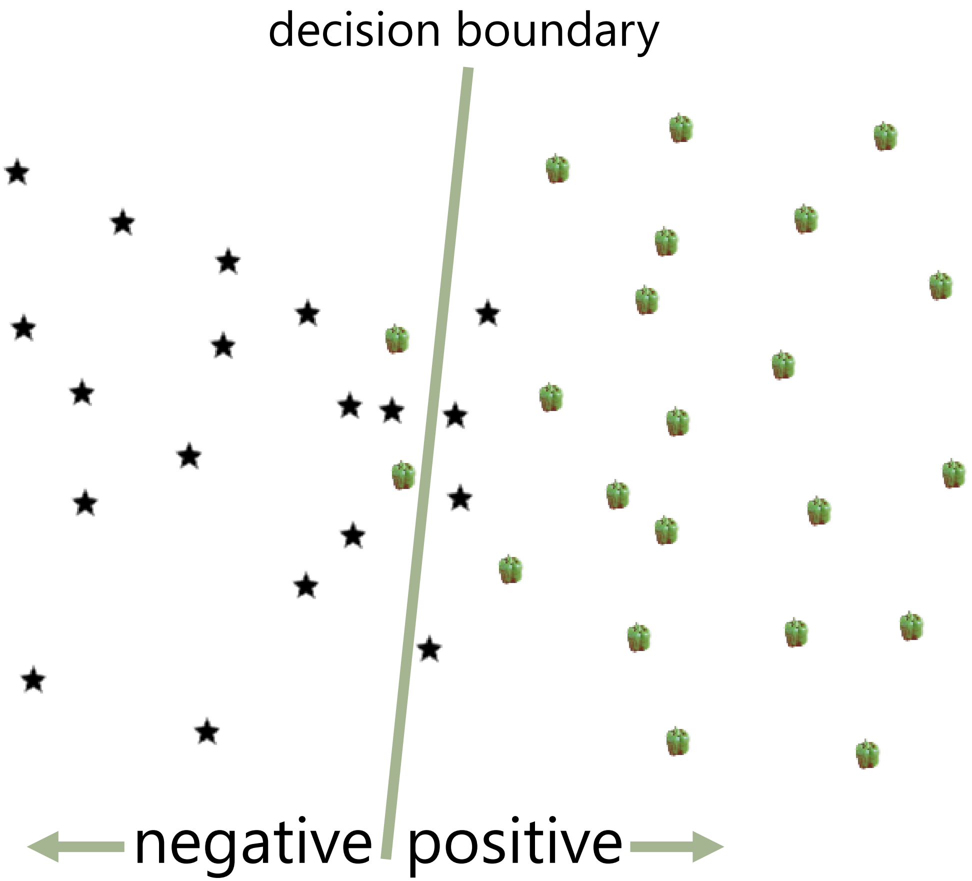
Which of these classifiers is best?

3. When we only care about the "most confident" predictions

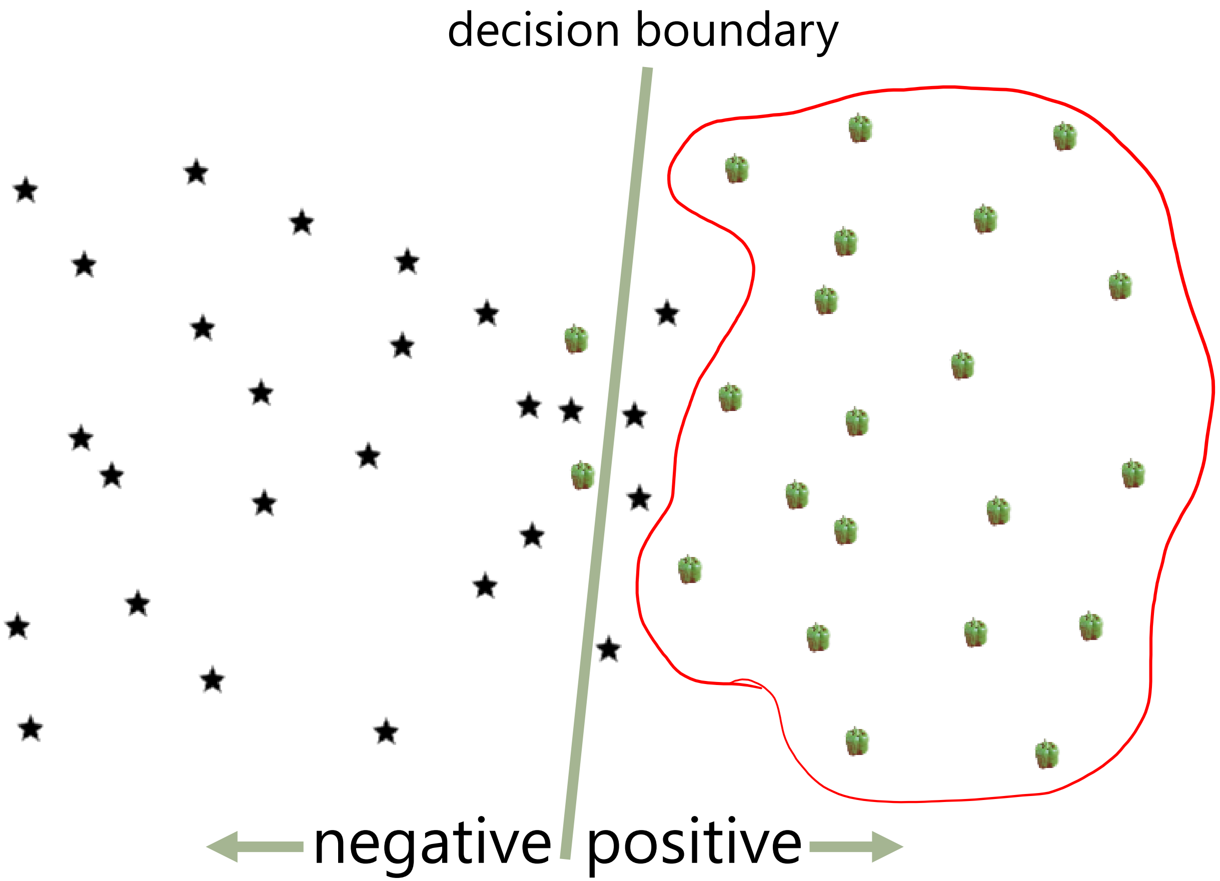
e.g. does a relevant result appear among the first page of results?



Evaluating classifiers

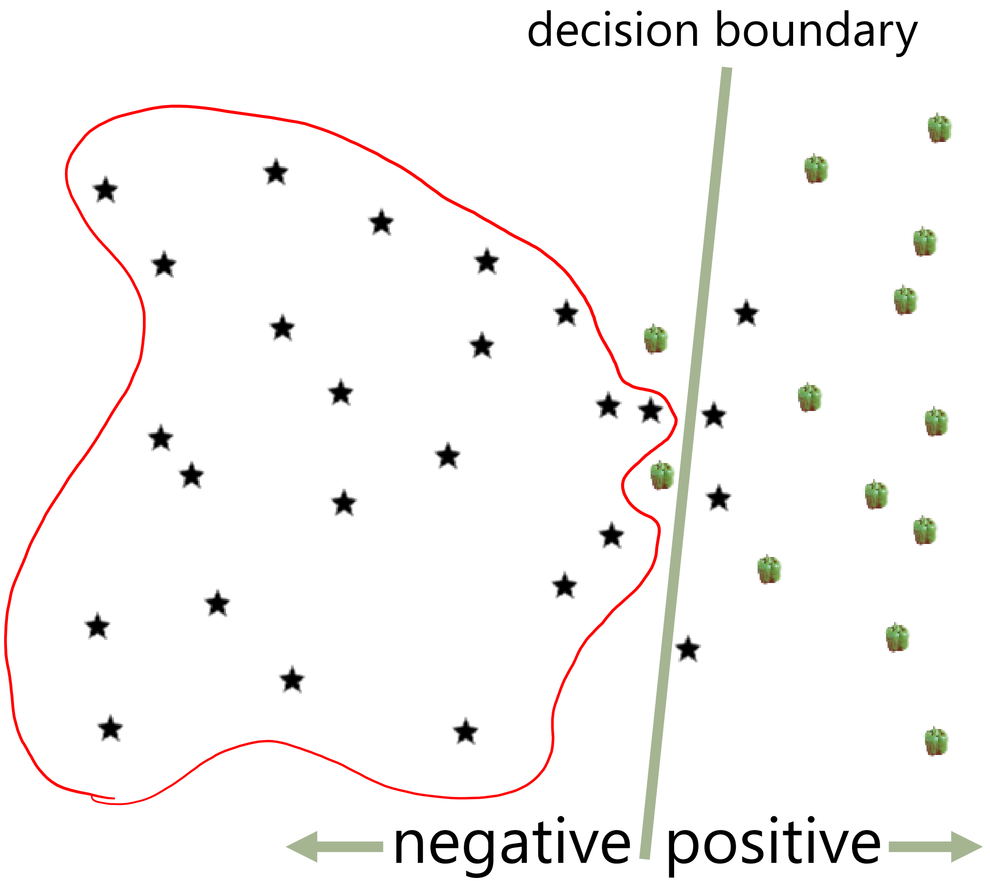


Evaluating classifiers



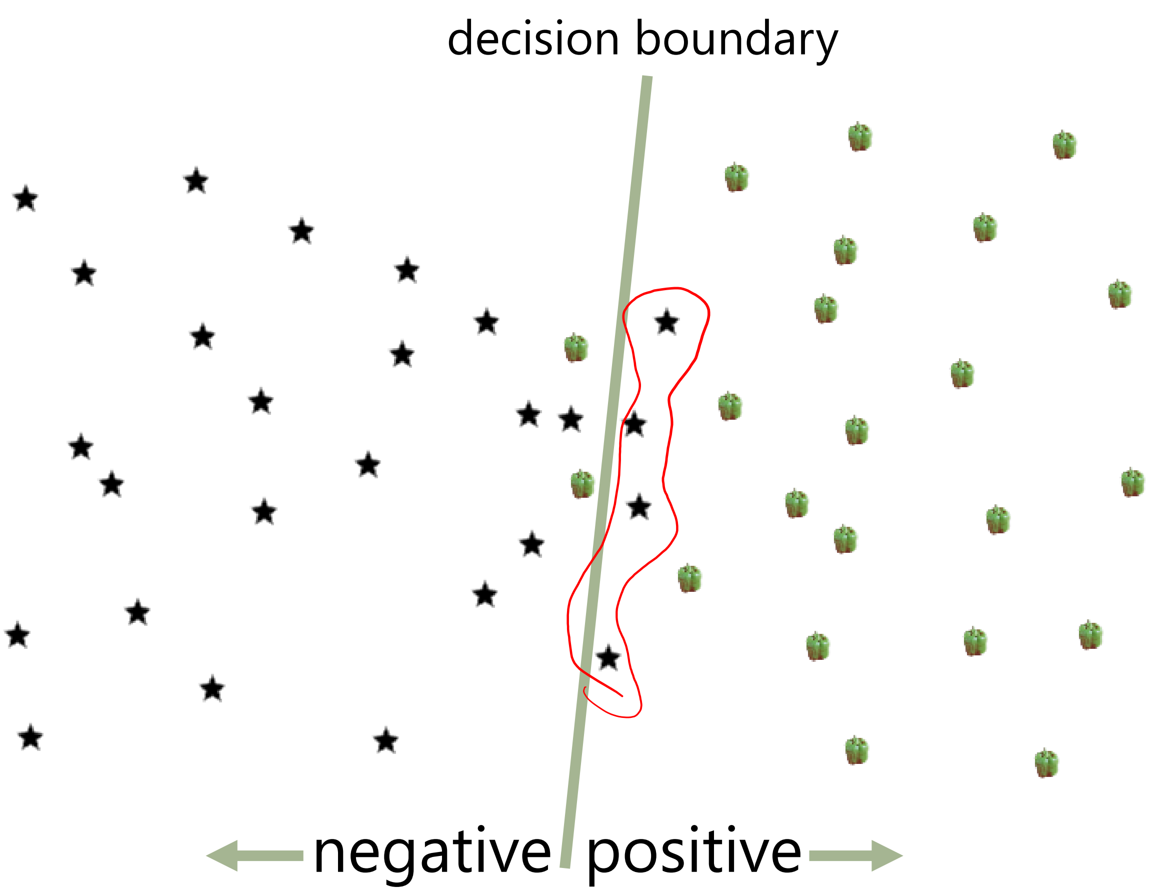
TP (true positive): Labeled as T, predicted as T

Evaluating classifiers



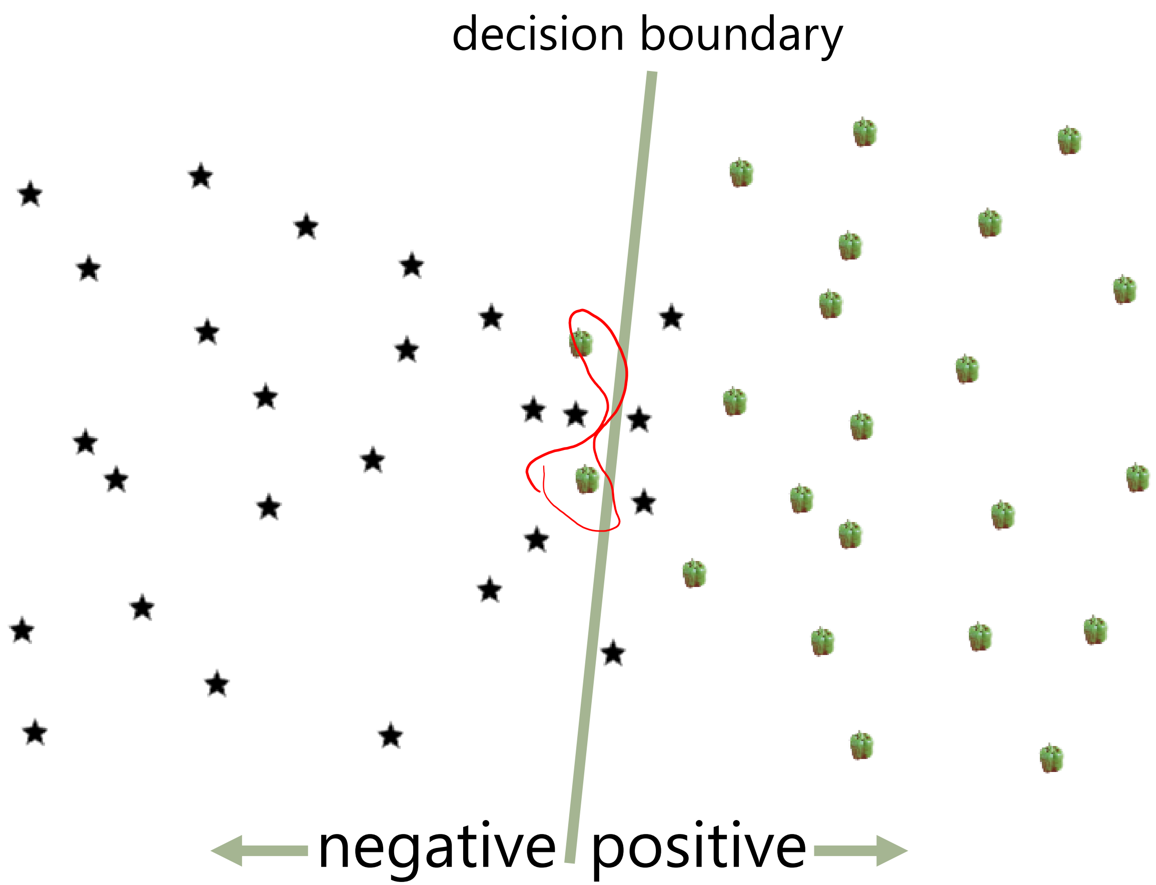
TN (true negative): Labeled as F , predicted as F

Evaluating classifiers



FP (false positive): Labeled as F , predicted as T

Evaluating classifiers



FN (false negative): Labeled as T , predicted as F

Evaluating classifiers

		Label	
		true	false
Prediction	true	true positive	false positive
	false	false negative	true negative

Classification accuracy = correct predictions / #predictions

$= (TP + TN) / (TP + TN + FP + FN)$

Error rate = 1 - accuracy = incorrect predictions / #predictions

$= (FP + FN) / (TP + TN + FP + FN)$

Evaluating classifiers

		Label	
		true	false
Prediction	true	true positive	false positive
	false	false negative	true negative

True positive rate (**TPR**) = true positives / #labeled positive

= TP / (TP + FN)

True negative rate (**TNR**) = true negatives / #labeled negative

= TN / (TN + FP)

Evaluating classifiers

		Label true	**Label** false
Prediction	true	true positive	false positive
	false	false negative	true negative

Balanced Error Rate (BER) = ½ (FPR + FNR)

$= 1 - \frac{1}{2}(TPR + TNR)$

= ½ for a random/naïve classifier, 0 for a perfect classifier

Evaluating classifiers

e.g.

y = [1, -1,   1,   1, 1, -1, 1, 1, -1, 1]

Confidence = [1.3,-0.2,-0.1,-0.4,1.4,0.1,0.8,0.6,-0.8,1.0]

TP TN FN FN TP FP TP TP TN TP

$\theta \cdot x_i$

$$TPR = TP / (TP + FN) = 5/7$$

$$TNR = TN / (TN + FP) = 2/3$$

$$BER = 1 - \frac{1}{2}\left(\frac{5}{7} + \frac{2}{3}\right)$$

Evaluating classifiers

How to optimize a balanced error measure:

$$L_\theta(y|X) = \prod_{y_i=1} p_\theta(y_i|X_i) \prod_{y_i=0} (1 - p_\theta(y_i|X_i))$$

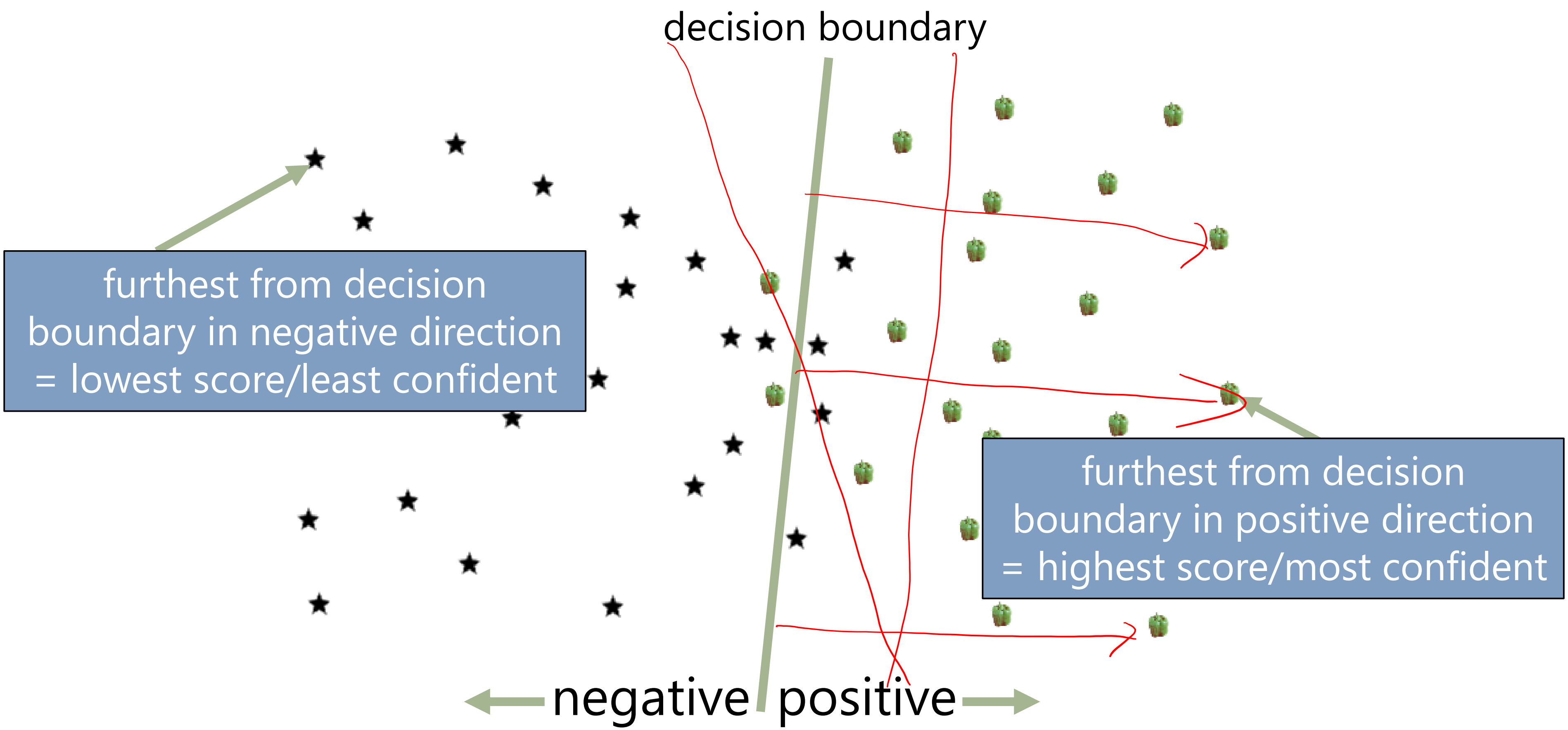
$$\ell(y|X) = \sum_{y_i=1} \log \sigma(\theta \cdot x_i) + \sum_{y_i=0} \log(1 - \sigma(\theta \cdot x_i))$$

Balanced

$$= \frac{1}{|y_i=1|} \sum_{y_i=1} \log \sigma(\theta \cdot x_i) + \frac{1}{|y_i=0|} \sum_{y_i=0} \log(1 - \sigma(\theta \cdot x_i))$$

Evaluating classifiers – ranking

The classifiers we've seen can associate **scores** with each prediction



Evaluating classifiers – ranking

The classifiers we've seen can associate **scores** with each prediction

- In ranking settings, the actual labels assigned to the points (i.e., which side of the decision boundary they lie on) **don't matter**
- All that matters is that positively labeled points tend to be at **higher ranks** than negative ones

Evaluating classifiers – ranking

The classifiers we've seen can associate **scores** with each prediction

- For naïve Bayes, the "score" is the ratio between an item having a positive or negative class
- For logistic regression, the "score" is just the probability associated with the label being 1 $\sigma(X;\theta)$
- For Support Vector Machines, the score is the distance of the item from the decision boundary (together with the sign indicating what side it's on)

Evaluating classifiers – ranking

The classifiers we've seen can associate **scores** with each prediction

e.g.

```
         y = [  1,  -1,   1,   1,  1, -1,  1,  1,  -1,  1]
Confidence = [1.3,-0.2,-0.1,-0.4,1.4,0.1,0.8,0.6,-0.8,1.0]
```

$\theta \cdot x_i$

Sort **both** according to confidence:

[1.4, 1.3, 1.0, 0.8, 0.6, 0.1, -0.1, -0.2, -0.4, -0.8]

[1 1 1 1 1 -1 1 -1 1 -1]

Evaluating classifiers – ranking

The classifiers we've seen can associate **scores** with each prediction

Labels sorted by confidence:

$$[\underbrace{1, 1, 1, 1, 1, -1}, 1, -1, 1, -1]$$

Suppose we have a fixed budget (say, six) of items that we can return
(e.g. we have space for six results in an interface)

- Total number of **relevant** items = 7
- Number of items we returned = 6
- Number of **relevant items** we returned = 5

Evaluating classifiers – ranking

The classifiers we've seen can associate **scores** with each prediction

$$\text{precision} = \frac{|\{\text{relevant documents}\} \cap \{\text{retrieved documents}\}|}{|\{\text{retrieved documents}\}|}$$

$\frac{5}{6}$

"fraction of retrieved documents that are relevant"

$$\text{recall} = \frac{|\{\text{relevant documents}\} \cap \{\text{retrieved documents}\}|}{|\{\text{relevant documents}\}|}$$

$\frac{5}{7}$

"fraction of relevant documents that were retrieved"

Evaluating classifiers – ranking

The classifiers we've seen can associate **scores** with each prediction

$\mathrm{precision}@k$ = precision when we have a budget of k retrieved documents

e.g.

- Total number of **relevant** items = 7
- Number of items we returned = 6
- Number of **relevant items** we returned = 5

precision@6 = $\frac{5}{6}$

Evaluating classifiers – ranking

The classifiers we've seen can associate **scores** with each prediction

$$F_1 = 2 \cdot \frac{\text{precision} \cdot \text{recall}}{\text{precision} + \text{recall}}$$

(harmonic mean of precision and recall)

$$F_\beta = (1 + \beta^2) \cdot \frac{\text{precision} \cdot \text{recall}}{\beta^2 \text{precision} + \text{recall}}$$

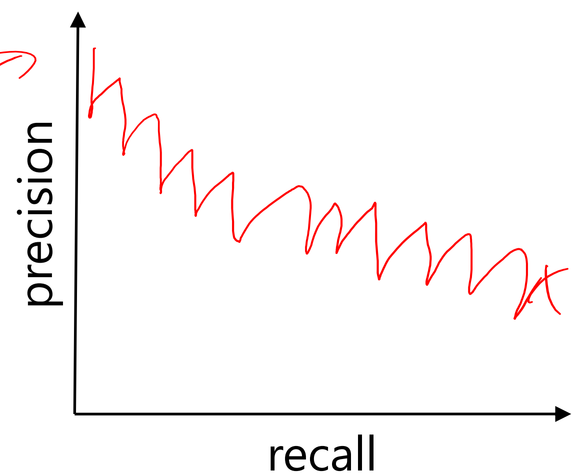
(weighted, in case precision is more important (low beta), or recall is more important (high beta))

Precision/recall curves

How does our classifier behave as we "increase the budget" of the number retrieved items?

- For budgets of size 1 to N, compute the precision and recall
- Plot the precision against the recall

only retrieved most confident prediction



fraction of relevant instances

retrieved everything

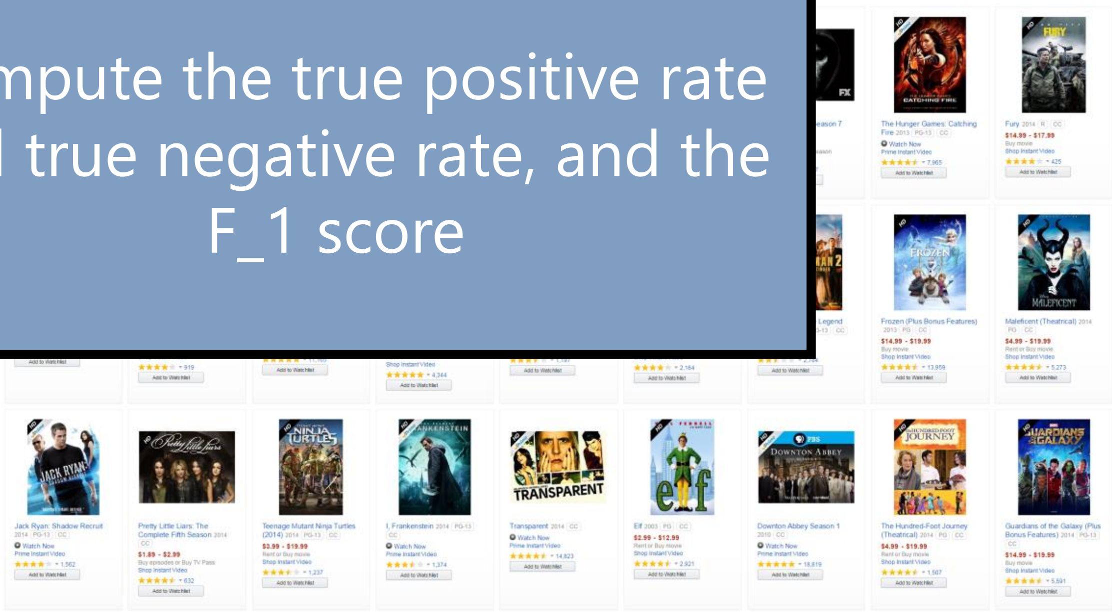
Summary

1. When data are highly imbalanced

If there are far fewer positive examples than negative examples we may want to assign additional weight to negative instances (or vice versa)

e.g. will I purchase
product? If I
purchase 0.00001
of products, then a
classifier which jus
predicts "no"
everywhere is
99.99999% accurate,
but not very useful

Compute the true positive rate and true negative rate, and the F_1 score



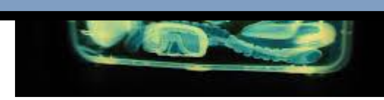
Summary

2. When mistakes are more costly in one direction

False positives are nuisances but false negatives are disastrous (or vice versa)

Compute “weighted” error measures that trade-off the precision and the recall, like the F_\beta score






e.g. which of these bags contains a weapon?

Summary

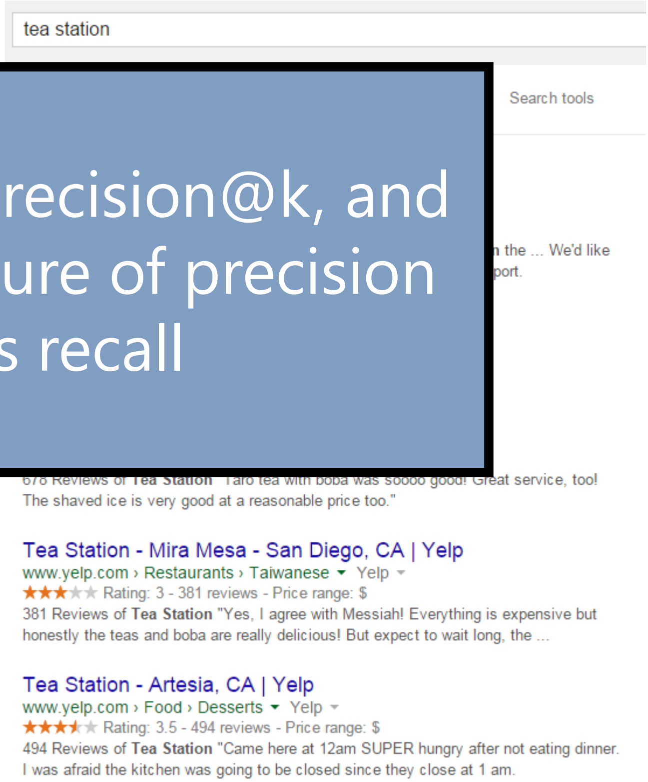
3. When we only care about the "most confident" predictions

e.g. does

result

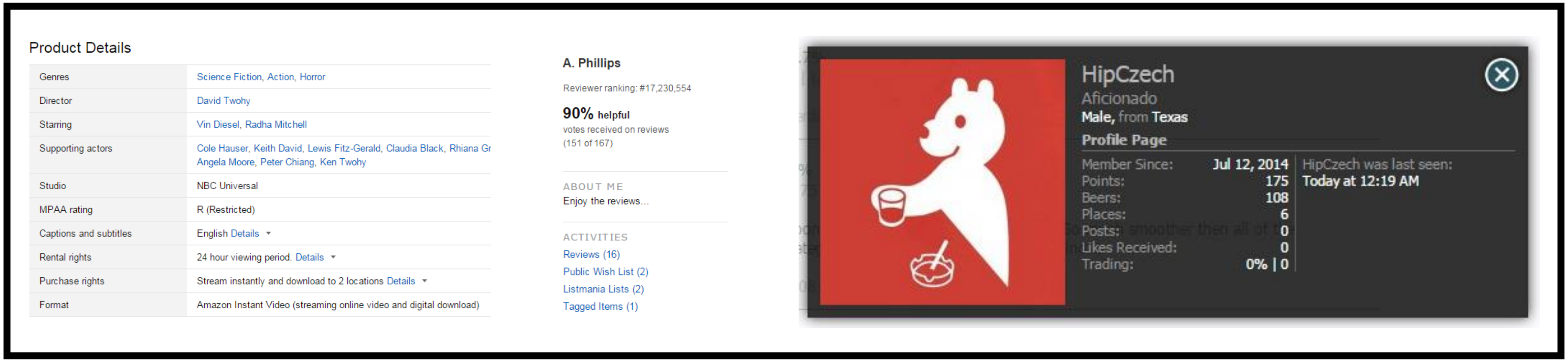
among

page of results?



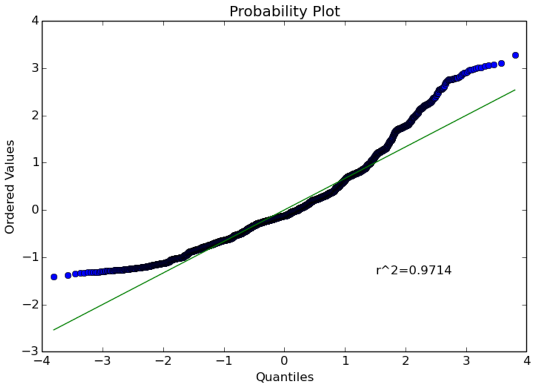
Compute the precision@k, and plot the signature of precision versus recall

So far: Regression



How can we use **features** such as product properties and user demographics to make predictions about **real-valued** outcomes (e.g. star ratings)?

How can we prevent our models from **overfitting** by favouring simpler models over more complex ones?



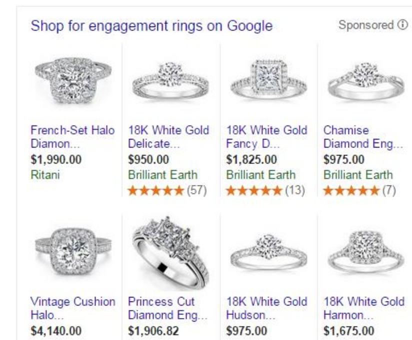
How can we assess our decision to optimize a particular error measure, like the MSE?

So far: Classification

Next we adapted these ideas to **binary** or **multiclass** outputs

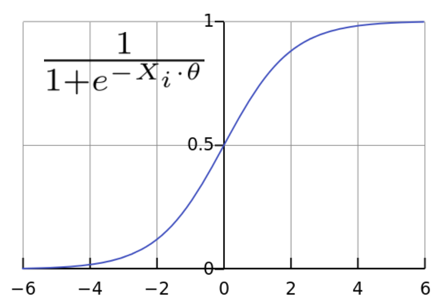
What animal is in this image?

Will I **purchase** this product?

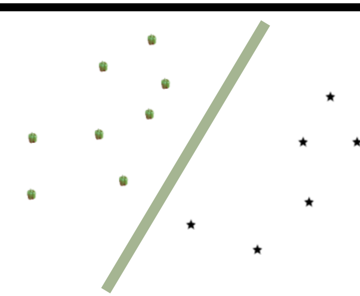


Will I **click on** this ad?

Combining features using naïve Bayes models



Logistic regression



Support vector machines

So far: supervised learning

Given **labeled training data** of the form

$$\{(\text{data}_1, \text{label}_1), \ldots, (\text{data}_n, \text{label}_n)\}$$

Infer the function

$$f(\text{data}) \stackrel{?}{\rightarrow} \text{labels}$$

So far: supervised learning

We've looked at two types of prediction algorithms:

Regression

$$y_i = X_i \cdot \theta$$

Classification

$$y_i = \begin{cases} 1 & \text{if } X_i \cdot \theta > 0 \\ 0 & \text{otherwise} \end{cases}$$

Questions?

Further reading:

- "Cheat sheet" of performance evaluation measures: http://www.damienfrancois.be/blog/files/modelperfcheatsheet.pdf
- Andrew Zisserman's SVM slides, focused on computer vision: http://www.robots.ox.ac.uk/~az/lectures/ml/lect2.pdf