1. Simplify the expression using K-Maps.
   (a) \( F(a, b, c) = \Sigma(1, 2, 3, 6, 7) \)
   (b) \( ab + a'b'c' + a'bc' \)
   Solution:
   (a) \( b + a'c \)
   (b) \( ab + a'c' \)

2. (a) Simplify the following expressions to minimal sum-of-products form using boolean algebra. Verify using K-Map. \((a' + c + d)(b + c + d)(a + b + c')\) HINT: Use consensus theorem.
   (b) Simplify the following expressions to minimal product-of-sums form using boolean algebra. Verify using K-Map. \(a'c + a'b'd + cd'\).
   NOTE: You do not need to mention the name of any theorem or axiom used.

Solution: (a) \( \text{Ans} = ac + c'd + a'b \)
\[ F = (a' + c + d)(b + c + d)(a + b + c') \]
\[ = (a'b + ac + ad + bc + cc + cd + bd + cd + dd)(a + b + c') \]
\[ = (a'b + ac + ad + bc + c + cd + bd + cd + d)(a + b + c') \]
\[ = (a'b + c(1 + a + b + d) + d(1 + a + b))(a + b + c') \]
\[ = (a'b + c + d)(a + b + c') \]
\[ = a'ab + a'bb + a'bc' + ac + bc + cc' + ad + bd + c'd \]
\[ = a'b + a'bc' + ac + bc + ad + bd + c'd \]
\[ = a'b + a'bc' + ac + ad + c'd \quad \ldots \text{using consensus thm} \]
\[ = a'b + a'bc' + ac + ad + c'd \quad \ldots \text{using consensus thm} \]
\[ a' b (1 + c') + ac + ad + c'd \]
\[ = a'b + ac + ad + c'd \]
\[ = a'b + ac + c'd \] \text{ ...using consensus thm} \\
\[ = ac + a'b + c'd \]

Minimum cover is the same as answer.

(b) \( \text{Ans} = (b' + c)(c + d)(a' + d') \)

\[ F = a'c + a'b'd + cd' \]
\[ F' = (a'c + a'b'd + cd')' \]
\[ = (a + c')(a + b+d')(c' + d) \]
\[ = (a + bc' + c'd')(c' + d) \]
\[ = ac' + ad + bc' + c'd' \]
\[ = ac' + ad + bc' + c'd' \]
\[ = ad + bc' + c'd' \text{ using consensus thm} \]

Hence, \( F = (a' + d')(b' + c)(c + d) \)
3. Logic minimisation using k-maps
   a. A bulb in the staircase has 3 switches. The bulb is turned ON for the following state of the switches.
      - Switch A is ON, Switches B,C are OFF
      - Switch C is ON, Switches A,B are OFF
      - Switches B,C are ON, Switch A is OFF
      - Switches A,C are ON, Switch B is OFF
   Answer the following questions for the above problem
   i. Give the truth table
   ii. Give expression for output in canonical sum of products form
   iii. Minimize the output expression obtained in part ii using k-map
   iv. Draw the logic circuit for the minimized equation from part iii using any logic gates

b. Determine the minimized SOP and POS expression of the following function using k-maps
   \[ f(A, B, C, D) = \sum m(4, 6, 8, 10, 11, 15) + \sum d(3, 5, 7, 9) \]

Solution:
a. Word problem
   i. Let the three switches be A,B,C and the output light bulb be Y

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ii. Output expression in canonical sum of product form
\[ Y = \overline{A} \overline{B} C + \overline{A} B C + A \overline{B} \overline{C} + A B C \]

iii. Minimisation using k-maps

\[ Y = A \overline{B} + \overline{A} C \]

iv. Logic gate implementation

b. K-map SOP and POS

SOP minimisation
\[ f(A, B, C, D) = \overline{A} B + A \overline{B} + C D \]

POS minimisation
Determine whether the two following circuits are equivalent using Boolean algebra.

Circuit 1:

\[ f(A, B, C, D) = (A + B) (C + \overline{D}) (\overline{A} + \overline{B} + D) \]

\[ f = ab(c+d') \]

Circuit 2:

\[ f = abc + abd' \quad \text{(de Morgan)} \]

\[ f = ab(c + d') \rightarrow \text{equivalent} \]

Alternative Solution: This aforementioned not gate can be presented as a bubble on the second to last AND gate, which has bubbled inputs. We can convert this bubbled AND gate into an OR, and simplify the circuit:
We can now write the expression for $f$ using this simplified circuit:

$$F = ab \cdot c + ab \cdot d' = ab(c + d')$$

→ Two circuits are equivalent