Recall terminology

- **Alphabet**: nonempty finite set of symbols
- **String over an alphabet**: finite sequence of symbols
- **Language over an alphabet**: set of strings

- **DFA over an alphabet**: deterministic finite automaton
  - Input: finite string over a fixed alphabet
  - Output: "accept" or "reject"
  - $L(M) = \{w \mid M \text{ accepts } w\}$

![DFA Diagram]

Start state (triangle/arrow)
Accept state (double circle)
An example

What's the best description of the language recognized by this DFA?

A. Starts with b and ends with a or b
B. Starts with a and ends with a or b
C. a's followed by b's
D. More than one of the above
E. I don't know.

and using set notation?
An example

This DFA recognizes the language of all strings of the form a's followed by b's

i.e. \{ a^n b^k \mid n,k \geq 1 \}
A language is **regular** if there is some finite automaton that recognizes **exactly** it.

If $A$ is the set of strings that DFA $M$ recognizes (accepts)

- We say $A$ is the **language** of $M$
- We write $L(M) = A$
- We conclude that $A$ is **regular**
An example

The language of all strings of the form $a$'s followed by $b$'s

i.e. $\{ a^n b^k | n, k \geq 1 \}$

is a regular language since it is recognized by this DFA.
Another example

What is the best description of the regular language recognized by this automaton?

A. \{ a^n b^k \mid n,k \geq 1 \}
B. \{ a^n b^k \mid n \geq 1, k \geq 0 \}
C. \{ awb \mid w \text{ in } \{a,b\}^* \}
D. \{ aw \mid w \text{ in } \{a,b\}^* \}
E. I don't know
Specifying an automaton

\( ( \{q1, q2, q3\}, \{a, b\}, \delta, q1, F ) \)

What state(s) should be in the set \( F \) so that the language of this machine is \( \{ w \mid \text{ab is a substring of } w \} \)?

A. \( F = \{q2\} \)
B. \( F = \{q3\} \)
C. \( F = \{q1, q2\} \)
D. \( F = \{q1, q3\} \)
E. I don't know.
Specifying an automaton

\((q_1, q_2, q_3), \{a, b\}, \delta, q_1, F\)  

What state(s) should be in the set \(F\) so that the language of this machine is \(\{ w | \text{b's never occur after a's in } w \}\)?

A. \(F = \{q_2\}\)  
B. \(F = \{q_3\}\)  
C. \(F = \{q_1, q_2\}\)  
D. \(F = \{q_1, q_3\}\)  
E. I don't know.
Regular languages: general facts

Is there an infinite regular language?

A. No: all regular languages have to be finite.
B. Yes: all regular sets are infinite.
C. Yes: all infinite sets of strings over an alphabet are regular.
D. Yes: some infinite sets of strings over each alphabet are regular and some are not.
E. I don't know.
Is every finite language regular?

A. No: some finite languages are regular, and some are not.
B. No: there are no finite regular languages.
C. Yes: every finite language is regular.
D. I don't know.
Regular languages: general facts

True/ False: each DFA recognizes a unique language. I.e. if two DFA are different (different number of states or different initial state, or different transition function, etc.) then they recognize different languages.

A. True  can you prove it?
B. False  can you prove it?
C. I don't know.
A useful (optional) bit of terminology

When is a string accepted by a DFA?

**Computation of $M$ on $w$:** where do we land when start at $q_0$ and read each symbol of $w$ one-at-a-time?

$$\delta^*(q_0, w) =$$
Building DFA

**Typical questions:**

Define a DFA which recognizes the given language L.

or

Prove that the (given) language L is regular.
Building DFA

Example
Define a DFA which recognizes

\{ w \mid w \text{ has at least 2 a's}\}
Building DFA

Example
Define a DFA which recognizes
\{ w \mid w \text{ has at most 2 a's}\}
Building DFA

Remember

States are our only (computer) memory.

Design and pick states with specific roles / tasks in mind.

"Have not seen any of desired pattern yet"
"Trap state"
The regular operations  

For A, B languages over same alphabet, define:

\[ A \cup B = \{ x | x \in A \text{ or } x \in B \} \]
\[ A \circ B = \{ xy | x \in A \text{ and } y \in B \} \]
\[ A^* = \{ x_1 x_2 \ldots x_k | k \geq 0 \text{ and each } x_i \in A \} \]

These are operations on sets!
Closure of \( \mathbb{Z} \) under addition.
Set of even ints under multiplication.
\( \{0\}^* \) under concatenation.

Which of these is true?

A. The set of odd integers is closed under addition.
B. The set of positive integers is closed under subtraction.
C. The set of rational numbers is closed under multiplication.
D. The set of real numbers is closed under division.
E. I don't know.