INSTRUCTIONS

Homework solutions should be neatly written or typed and submitted through Gradescope. No work can be accepted outside of this system, and no late work will be accepted. Please ensure that your submission is legible (neatly written and not too faint) or your homework may not be graded. You may update your submission as many times as you’d like up to the deadline. Only the most recent submission will be graded.

Thirty problems from homework assignments will be graded randomly throughout the quarter. You will not know in advance which problems, if any, will be graded on each assignment.

You may consult your textbook, class notes, lecture slides, instructors, TAs, and tutors for help with homework. You may also discuss homework questions with classmates, but you may not share written work with classmates. You must write up your solutions alone, in your own words. The assignments have been developed to facilitate your learning and to provide a method for fairly evaluating your knowledge and abilities, not the knowledge and abilities of others. To facilitate learning, you are authorized to discuss assignments with others; however, to ensure fair evaluations, you are not authorized to view or share written work with another person, or to write your submission in collaboration with another person. You should not look for answers to homework problems in other texts or sources, including the internet.

Do not post about homework questions on Piazza. For help with homework, please consult the course textbook, lecture slides, class notes, and podcasts, or come visit us in office hours.

READING: Sipser 1.4, 2.1
1. Consider the language $L$ of strings over the alphabet \{a, b\} such that the number of a’s minus the number of b’s is at least three. For example, abaaa, aaaaaa, aabaababaa are in $L$ but bb, aabbaa, aa are not in $L$. Fill in the missing parts of the following proof to show that $L$ is not regular.

Proof: Assume (towards a contradiction) that $L$ is regular. Then the Pumping Lemma applies to $L$. Let $p$ be the pumping length of $L$. Choose $s$ to be the string $abaaab$. Since this string is in $L$ and has length greater than or equal to $p$, the Pumping Lemma guarantees $s$ can be divided into parts $s = xyz$ such that for any $i \geq 0$, $xy^i z$ is in $L$, and that $|y| > 0$ and $|xy| \leq p$. But if we let $i = 2$, we get a string $xy^2 z$ that is not in $L$ because this is a contradiction. Therefore the assumption is false, and $L$ is not regular.

2. Consider the language $L = \{tu \mid t \text{ and } u \text{ are strings over } \{0, 1\} \text{ with the same number of 1’s}\}$. Explain and correct the error below:

Proof that $L$ is not regular using the Pumping Lemma:

Assume (towards a contradiction) that $L$ is regular. Then the Pumping Lemma applies to $L$. Let $p$ be the pumping length of $L$. Choose $s$ to be the string $1p0p1p0p$, which is in $L$ because $t = 1p0p$ and $u = 1p0p$ each have $p$ 1’s.

Since this string is in $L$ and has length greater than or equal to $p$, the Pumping Lemma guarantees $s$ can be divided into parts $s = xyz$ such that for any $i \geq 0$, $xy^i z$ is in $L$, and that $|y| > 0$ and $|xy| \leq p$. Since the first $p$ characters of $s$ are all 1’s and we have $|y| > 0$ and $|xy| \leq p$, we know that $y$ must be nonempty and made up of all 1’s. But if we let $i = 2$, we get a string $xy^2 z$ that is not in $L$ because repeating $y$ twice adds 1’s to $t$ but not to $u$, and strings in $L$ are required to have the same number of 1’s in $t$ as in $u$.

This is a contradiction. Therefore the assumption is false, and $L$ is not regular.

3. Give a context-free grammar to generate each language below.

(a) $L_1 = \{w \in \{0, 1\}^* \mid \text{the length of } w \text{ is odd}\}$
(b) $L_2 = \{w \in \{0, 1\}^* \mid \text{the length of } w \text{ is odd and } w \text{ starts with } 1\}$
(c) $L_3 = \{w \in \{0, 1\}^* \mid \text{the length of } w \text{ is odd and } w \text{ starts and ends with } 1\}$
(d) $L_4 = \{w \in \{0, 1\}^* \mid \text{the length of } w \text{ is odd and the middle symbol of } w \text{ is } 1\}$. 