Instructions

Homework solutions should be neatly written or typed and submitted through Gradescope. No work can be accepted outside of this system, and no late work will be accepted. Please ensure that your submission is legible (neatly written and not too faint) or your homework may not be graded. You may update your submission as many times as you’d like up to the deadline. Only the most recent submission will be graded.

Thirty problems from homework assignments will be graded randomly throughout the quarter. You will not know in advance which problems, if any, will be graded on each assignment.

You may consult your textbook, class notes, lecture slides, instructors, TAs, and tutors for help with homework. You may also discuss homework questions with classmates, but you may not share written work with classmates. You must write up your solutions alone, in your own words. The assignments have been developed to facilitate your learning and to provide a method for fairly evaluating your knowledge and abilities, not the knowledge and abilities of others. To facilitate learning, you are authorized to discuss assignments with others; however, to ensure fair evaluations, you are not authorized to view or share written work with another person, or to write your submission in collaboration with another person. You should not look for answers to homework problems in other texts or sources, including the internet.

Do not post about homework questions on Piazza. For help with homework, please consult the course textbook, lecture slides, class notes, and podcasts, or come visit us in office hours.

Reading: Sipser 1.2, 1.3
1. Give a counterexample to show that the following construction does not prove the closure of the regular languages under the star operation. That is, you must give an example of an NFA $N$ for which the constructed automaton $\hat{N}$ does not recognize the star of $N$’s language. Draw your automaton $N$ and say why it serves as a counterexample to the construction below.

Construction: Let $N = (Q, \Sigma, \delta, q_0, F)$ be an NFA. Construct an NFA $\hat{N} = (Q, \Sigma, \hat{\delta}, q_0, \tilde{F})$ as follows. $\hat{N}$ is supposed to recognize $L(N)^*$.

- The states of $\hat{N}$ are the states of $N$.
- The alphabet of $\hat{N}$ is the alphabet of $N$.
- The start state $q_0$ of $\hat{N}$ is the same as the start state of $N$.
- $\tilde{F} = F \cup \{q_0\}$. The accept states of $\hat{N}$ are the accept states of $N$, along with the start state.
- Define $\hat{\delta}$ so that for any $q \in Q$ and any $a \in \Sigma$,

$$\hat{\delta}(q, a) = \begin{cases} \delta(q, a) \cup \{q_0\}, & \text{if } q \in F \text{ and } a = \epsilon \\ \delta(q, a), & \text{otherwise}. \end{cases}$$

2. Let $\Sigma = \{a, b\}$. For each regular expression, describe the language of the regular expression by choosing one of the sets named below, or saying “none” if the language of the regular expression is not any of the given sets. Sets may be used more than once or not at all.

Sets:
- $A = \{w \in \Sigma^* \mid w \text{ does not contain the substring } bb\}$
- $B = \{w \in \Sigma^* \mid w \text{ contains the substring } bb\}$
- $C = \{w \in \Sigma^* \mid w \text{ starts or ends with } bb\}$
- $D = \{w \in \Sigma^* \mid w \text{ starts and ends with } bb\}$
- $E = \{w \in \Sigma^* \mid w \text{ does not end with } bb\}$
- $F = \{w \in \Sigma^*\}$

Regular expressions:
- (a) $bb(a \cup b)^*bb$
- (b) $(ba \cup a)^* \cup (ba \cup a)^*b$
- (c) $(ba^* \cup a)^*$
- (d) $bb(a \cup b)^* \cup (a \cup b)^*bb$
- (e) $bb(a \cup b)^*bb \cup bb \cup bbb$
- (f) $b^*b^*(a \cup b)^* \cup (a \cup b)^*b^*b^*$
- (g) $(bb)^*$
- (h) $(a \cup ab)^* \cup b(a \cup ab)^*$
- (i) $(a \cup b)^*(a \cup ab) \cup \epsilon \cup b$
- (j) $(a^* \cup b^*)^*bb(a \cup b)^*$